

# Reasoning for Humans: Clear Thinking in an Uncertain World

PHIL 171

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# Kolmogorov Axioms

Given any stochastic truth table, for any formulas  $\varphi$  and  $\psi$ :

- $Pr(\varphi) \geq 0$
- If  $\varphi$  is a tautology, then  $Pr(\varphi) = 1$
- If  $\varphi$  and  $\psi$  are mutually exclusive, then  $Pr(\varphi \vee \psi) = Pr(\varphi) + Pr(\psi)$

# Laws of Probability

In any stochastic truth table, for all  $\varphi$ ,  $Pr(\neg\varphi) = 1 - Pr(\varphi)$

In any stochastic truth table, for all  $\varphi$ , if  $\varphi$  is a contradiction, then  $Pr(\varphi) = 0$

In any stochastic truth table, for all  $\varphi$  and  $\psi$ , if  $\varphi \leftrightarrow \psi$  is a tautology (i.e.,  $\varphi$  and  $\psi$  are tautologically equivalent), then  $Pr(\varphi) = Pr(\psi)$

# Laws of Probability

In any stochastic truth table, for all  $\varphi$  and  $\psi$ ,  
 $Pr(\varphi) = Pr(\varphi \wedge \psi) + Pr(\varphi \wedge \neg\psi)$

In any stochastic truth table, for all  $\varphi$  and  $\psi$ ,  
 $Pr(\varphi \vee \psi) = Pr(\varphi) + Pr(\psi) - Pr(\varphi \wedge \psi)$

In any stochastic truth table, for all  $\varphi$  and  $\psi$ ,  
if  $\varphi \rightarrow \psi$  is a tautology, then  $Pr(\varphi) \leq Pr(\psi)$

# Laws of Total Probability

In any stochastic truth table, for all  $\varphi$  and  $\psi$ ,

$$Pr(\varphi) = Pr(\psi)Pr(\varphi \mid \psi) + Pr(\neg\psi)Pr(\varphi \mid \neg\psi)$$

# Conjunctions and Conditional Probability

In any stochastic truth table, for all  $\varphi$  and  $\psi$ ,

$$Pr(\varphi \wedge \psi) = Pr(\varphi)Pr(\psi \mid \varphi)$$

In any stochastic truth table, for all  $\varphi$ ,  $\psi$ , and  $\chi$

$$Pr(\varphi \wedge \psi \mid \chi) = Pr(\varphi \mid \chi)Pr(\psi \mid \varphi \wedge \chi)$$

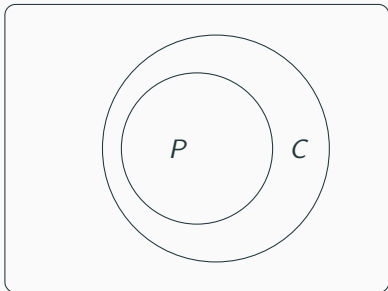
$$\frac{P}{\therefore C}$$

The argument is **deductively valid** when  $P \wedge \neg C$  is **impossible** (there is no truth-value function that makes  $P \wedge \neg C$  true).



$$\frac{P}{\therefore C}$$

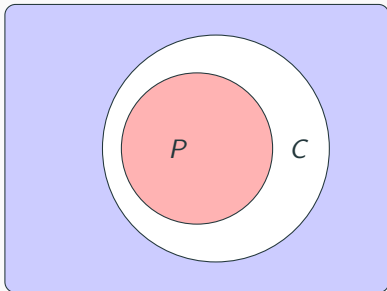
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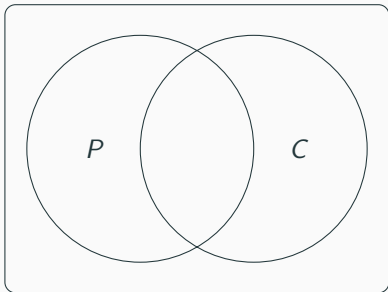
The argument  $P \Rightarrow C$  is (deductively) valid when there is no truth value assignment that makes  $P$  true and  $C$  false

The argument  $P \Rightarrow C$  is (deductively) valid when every truth value assignment makes  $P \rightarrow C$  true

The argument  $P \Rightarrow C$  is (deductively) valid when there is no truth value assignment that makes  $P \wedge \neg C$  true

$$\frac{P}{\therefore C}$$

The argument is **is inductively strong** if it is **improbable** that  $C$  is false **while**  $P$  is true (and it is not deductively valid).



$$\frac{P}{\therefore C}$$

When is an argument inductively strong?

...when it is *improbable* that its conclusion is false while its premises are true.

- when  $P \rightarrow C$  is probable?
- when  $P \wedge \neg C$  is improbable?

There is a 2000-year-old man in Cleveland.

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∴ There is a 2000-year-old man in Cleveland who has three heads.

This argument is *not* inductively strong, yet:

$P \wedge \neg C$  is improbable because  $P$  is improbable.

There is a man in Cleveland who is 1999 years and 11-months-old and in good health.

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$\therefore$  No man will live to be 2000 years old.

This argument is *not* inductively strong, yet:

$P \wedge \neg C$  is improbable because  $C$  is probable.

$$\frac{P}{\therefore C}$$

When is an argument inductively strong?

...when it is *improbable* that its conclusion is false while its premises are true.

- when  $P \rightarrow C$  is probable? **No!**
- when  $P \wedge \neg C$  is improbable? **No!**

The inductive probability of an argument is determined by the evidential relation between its premises and its conclusion, not by the likelihood of the truth of its premises alone or the likelihood of the truth of its conclusion alone.



Every day so far this semester, Ann brought her laptop to class.  
Therefore, Ann will bring her laptop to the next class.

Every day so far this semester, Ann brought her laptop to class.  
Therefore, Ann will bring her laptop to the next class.

Ann said that she saw Bob steal the laptop. Therefore, Bob stole the laptop

Ten eyewitnesses said that they saw Bob steal the laptop. Therefore, Bob stole the laptop.

# Conditional Probability

The inductive probability of an argument is the probability that its conclusion is true **given that its premises are true**.

$$Pr(C | P) = \frac{Pr(C \wedge P)}{Pr(P)}$$

There is a 2000-year-old man in Cleveland.

---

∴ There is a 2000-year-old man in Cleveland who has three heads.

This argument is **not** inductively strong:

$Pr(C | P)$  is low.

There is a man in Cleveland who is 1999 years and 11-months-old and in good health.

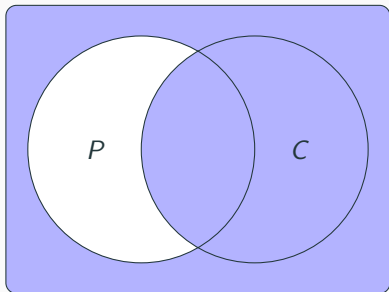
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∴ No man will live to be 2000 years old.

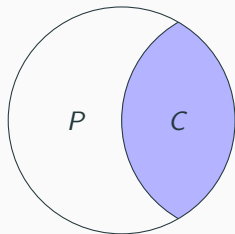
This argument is **not** inductively strong:

$Pr(C | P)$  is low.

$$\frac{P}{\therefore C}$$



$$Pr(P \rightarrow C)$$



$$Pr(C | P)$$

$$\frac{P}{\therefore C}$$

When is an argument inductively strong?

1.  $P$  **evidentially supports**  $C$ :  $Pr(C | P)$  is “high” (i.e.,  $Pr(C | P) > \frac{1}{2}$ ).
2. The argument is not valid.
3. Are we missing any other conditions?

Bob (who is male) is taking a birth control pill.

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∴ Bob (who is male) is not going to get pregnant.

$$Pr(C | P) = 1 = Pr(C)$$



Bob (who is male) is taking a birth control pill.

---

∴ Bob (who is male) is not going to get pregnant.

$$Pr(C | P) = 1 = Pr(C)$$

*P* is *irrelevant* to the truth of *C*

# Independence

Two formulas  $\varphi$  and  $\psi$  are **independent** given a stochastic truth table provided that  $Pr(\varphi) = Pr(\varphi \mid \psi)$ .

Equivalent ways of defining when two formulas  $\varphi$  and  $\psi$  are **independent** given a stochastic truth table:

- $Pr(\varphi \wedge \psi) = Pr(\varphi)Pr(\psi)$ .
- $Pr(\psi) = Pr(\psi \mid \varphi)$ .

## Example

Suppose that you are flipping two coins. Let  $P$  be true when the first coin landed heads and  $Q$  be true when the second coin landed heads.

## Example

Suppose you flip the coins many times and record the percentage of the four possible coin flips resulting in the following stochastic truth table:

	$P$	$Q$	$P \wedge Q$
0.25	T	T	T
0.25	T	F	F
0.25	F	T	F
0.25	F	F	F

$P$  and  $Q$  are independent:

- $Pr(P) = 0.5 = \frac{0.25}{0.5} = \frac{Pr(P \wedge Q)}{Pr(Q)} = Pr(P | Q)$
- $Pr(P \wedge Q) = 0.25 = 0.5 \times 0.5 = Pr(P) \times Pr(Q)$
- $Pr(Q) = 0.5 = \frac{0.25}{0.5} = \frac{Pr(Q \wedge P)}{Pr(P)} = Pr(Q | P)$

## Example

Suppose you flip the coins many times and record the percentage of the four possible coin flips resulting in the following stochastic truth table:

	$P$	$Q$	$P \wedge Q$
0.4	T	T	T
0.1	T	F	F
0.1	F	T	F
0.4	F	F	F

$P$  and  $Q$  are **not** independent:

- $Pr(P) = 0.5 \neq 0.8 = \frac{0.4}{0.5} = \frac{Pr(P \wedge Q)}{Pr(Q)} = Pr(P | Q)$
- $Pr(P \wedge Q) = 0.4 \neq 0.25 = 0.5 \times 0.5 = Pr(P) \times Pr(Q)$
- $Pr(Q) = 0.5 \neq 0.8 = \frac{0.4}{0.5} = \frac{Pr(Q \wedge P)}{Pr(P)} = Pr(Q | P)$

## Positive/Negative Relevance

$\varphi$  is **positively relevant** to  $\psi$  (given a stochastic truth table) when  
 $Pr(\psi | \varphi) > Pr(\psi)$

$\varphi$  is **negatively relevant** to  $\psi$  (given a stochastic truth table) when  
 $Pr(\psi | \varphi) < Pr(\psi)$

$$\frac{P}{\therefore C}$$

When is an argument inductively strong?

1.  $P$  **evidentially supports**  $C$ :  $Pr(C | P)$  is “high” (i.e.,  $Pr(C | P) > \frac{1}{2}$ ).
2.  $P$  is **positively relevant** to  $C$ :  $Pr(C | P) > Pr(C)$
3. The argument is not deductively valid.

# Inductive Strength

Argument 1

$$\frac{A \vee B}{\therefore A}$$

Argument 2

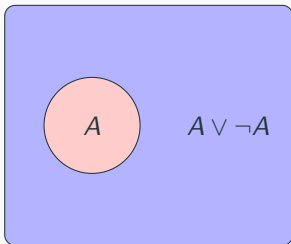
$$\frac{A \vee \neg A}{\therefore A}$$

- Argument 1: You will get either an A or B in PHIL 171. Therefore, you will get an A in PHIL 171.
- Argument 2: Either you will get an A or you will not get an A in PHIL 171. Therefore, you will get an A in PHIL 171.
- Argument 1 is not valid, and Argument 2 is not valid.
- Argument 1 is *stronger* than Argument 2.



# Inductive Strength

$$\frac{A \vee \neg A}{\therefore A}$$



$$\frac{A \vee B}{\therefore A}$$

