Reasoning for Humans: Clear Thinking in an Uncertain World

PHIL 171

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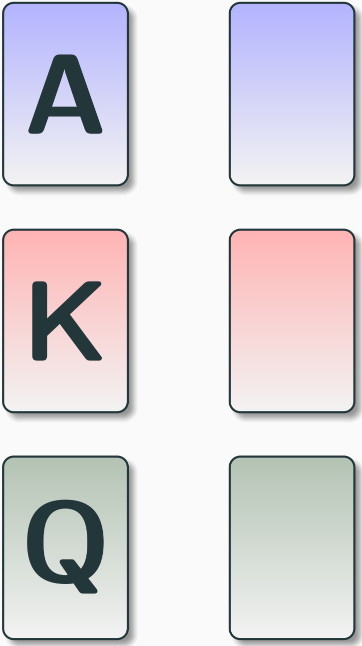
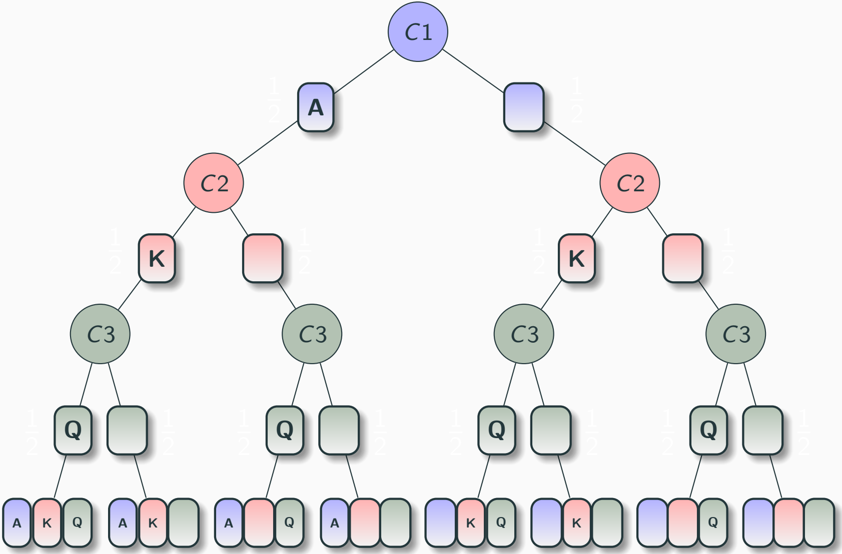
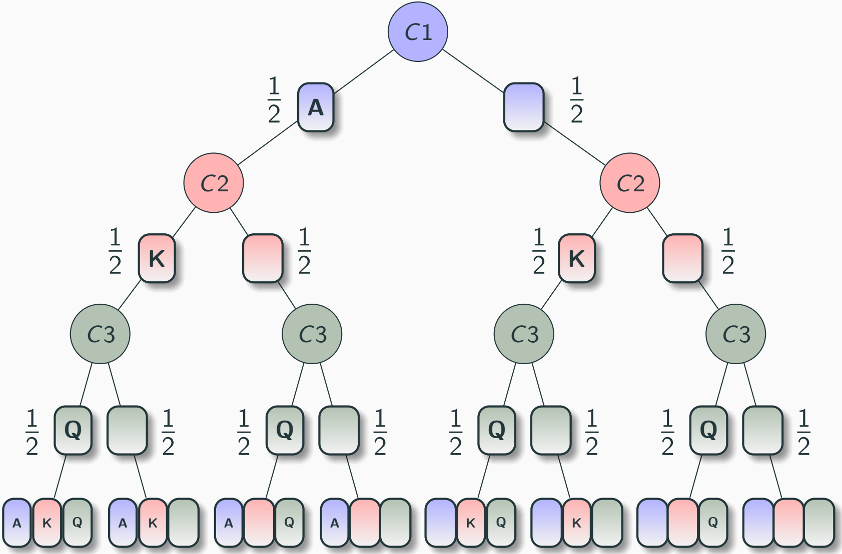
pacuit.org

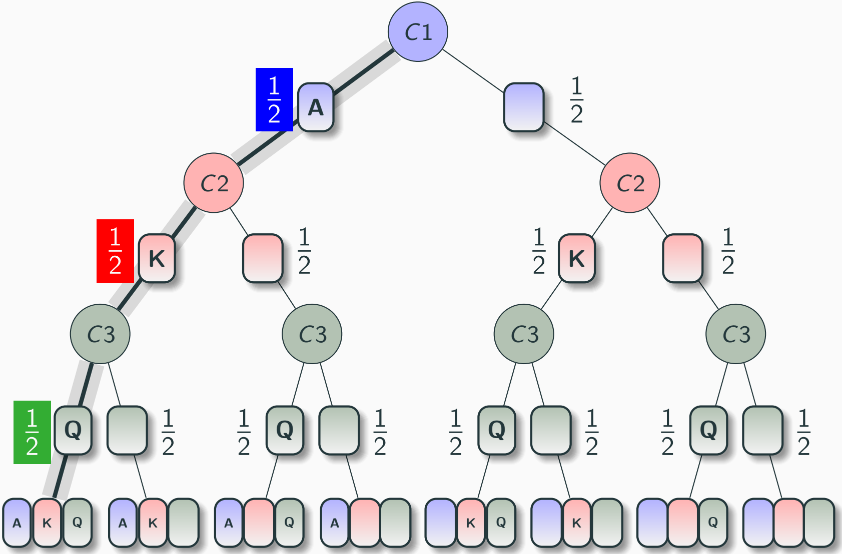

A
K
Q



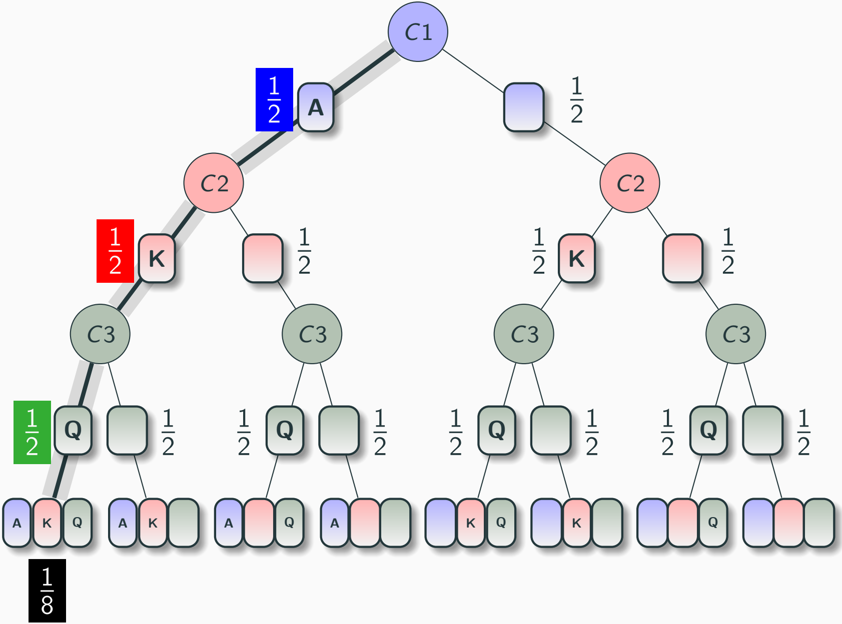
C1
A
1/2
1/2
C2
C2
K
1/2
1/2
K
1/2
1/2
C3
C3
C3
C3
Q
1/2
1/2
Q
1/2
1/2
Q
1/2
1/2
Q
1/2
1/2
A K Q
A K
A Q
A
K Q
K
Q

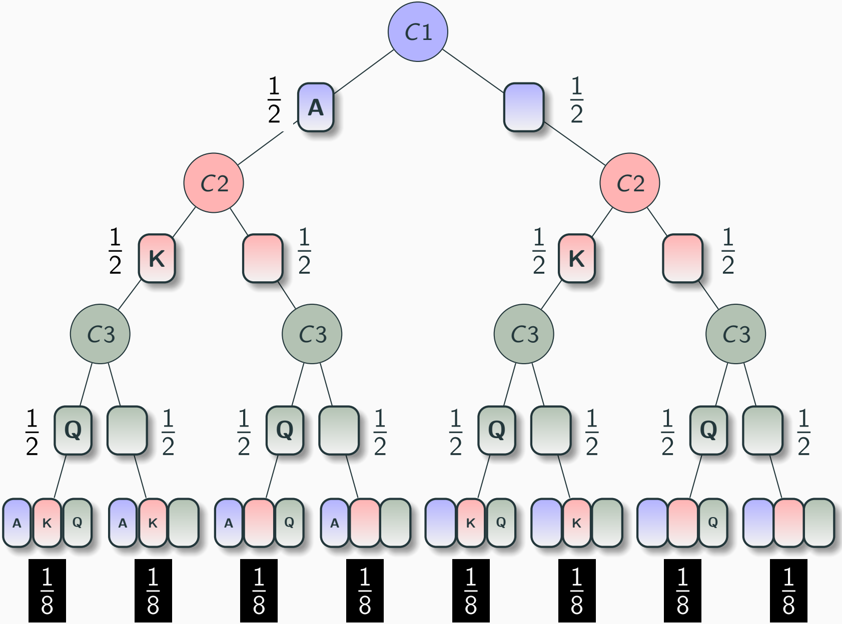


C1
1/2
A
1/2
C2
1/2
K
1/2
C2
1/2
K
1/2
C3
C3
C3
C3
1/2
Q
1/2
1/2
Q
1/2
1/2
Q
1/2
1/2
Q
1/2
A K Q
A K
A Q
A
K Q
K
Q



$$\frac{1}{2} * \frac{1}{2} * \frac{1}{2} = \frac{1}{8}$$


C1
1/2
A
1/2
C2
C2
1/2
K
1/2
1/2
K
1/2
C3
C3
C3
C3
1/2
Q
1/2
1/2
Q
1/2
1/2
Q
1/2
1/2
Q
1/2
A K Q
A K
A Q
A
K Q
K
Q
1/8


$C1$
$\frac{1}{2}$
A
$\frac{1}{2}$
$C2$
$C2$
$\frac{1}{2}$
K
$\frac{1}{2}$
$\frac{1}{2}$
K
$\frac{1}{2}$
$C3$
$C3$
$C3$
$C3$
$\frac{1}{2}$
Q
$\frac{1}{2}$
$\frac{1}{2}$
Q
$\frac{1}{2}$
$\frac{1}{2}$
Q
$\frac{1}{2}$
$\frac{1}{2}$
Q
$\frac{1}{2}$
A K Q
A K
A Q
A
K Q
K
Q
$\frac{1}{8}$
$\frac{1}{8}$
$\frac{1}{8}$
$\frac{1}{8}$
$\frac{1}{8}$
$\frac{1}{8}$
$\frac{1}{8}$
$\frac{1}{8}$

	A	K	Q	
	T	T	T	$A \wedge K \wedge Q$
	T	T	F	$A \wedge K \wedge \neg Q$
	T	F	T	$A \wedge \neg K \wedge Q$
	T	F	F	$A \wedge \neg K \wedge \neg Q$
	F	T	T	$\neg A \wedge K \wedge Q$
	F	T	F	$\neg A \wedge K \wedge \neg Q$
	F	F	T	$\neg A \wedge \neg K \wedge Q$
	F	F	F	$\neg A \wedge \neg K \wedge \neg Q$

prob	A	K	Q	
$\frac{1}{8}$	T	T	T	$A \wedge K \wedge Q$
$\frac{1}{8}$	T	T	F	$A \wedge K \wedge \neg Q$
$\frac{1}{8}$	T	F	T	$A \wedge \neg K \wedge Q$
$\frac{1}{8}$	T	F	F	$A \wedge \neg K \wedge \neg Q$
$\frac{1}{8}$	F	T	T	$\neg A \wedge K \wedge Q$
$\frac{1}{8}$	F	T	F	$\neg A \wedge K \wedge \neg Q$
$\frac{1}{8}$	F	F	T	$\neg A \wedge \neg K \wedge Q$
$\frac{1}{8}$	F	F	F	$\neg A \wedge \neg K \wedge \neg Q$

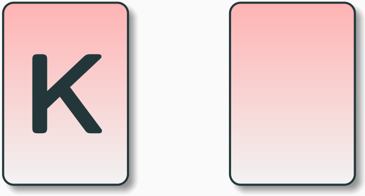
Biased dealer

- When selecting a red card, he is 3-times more likely to deal a K than a blank.

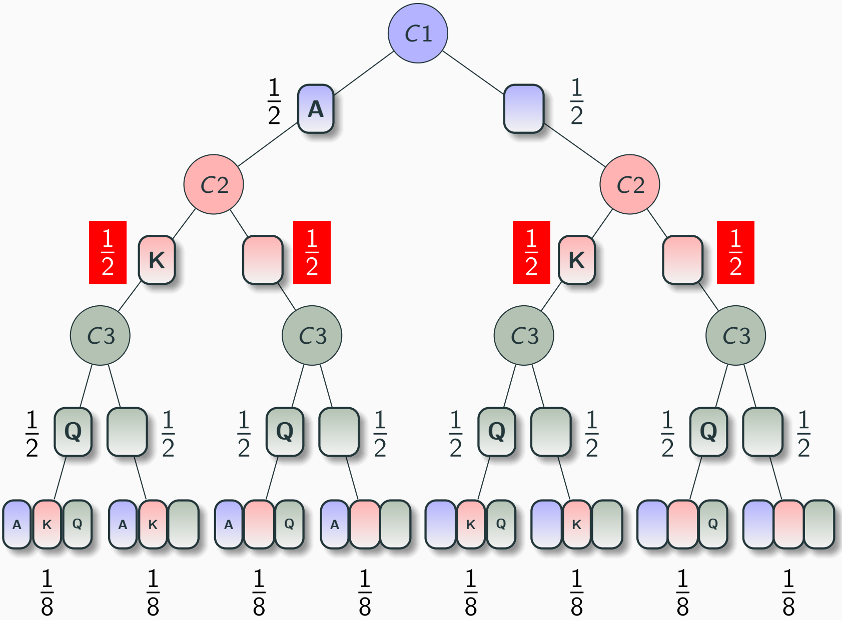
Biased dealer

- When selecting a red card, he is 3-times more likely to deal a K than a blank.

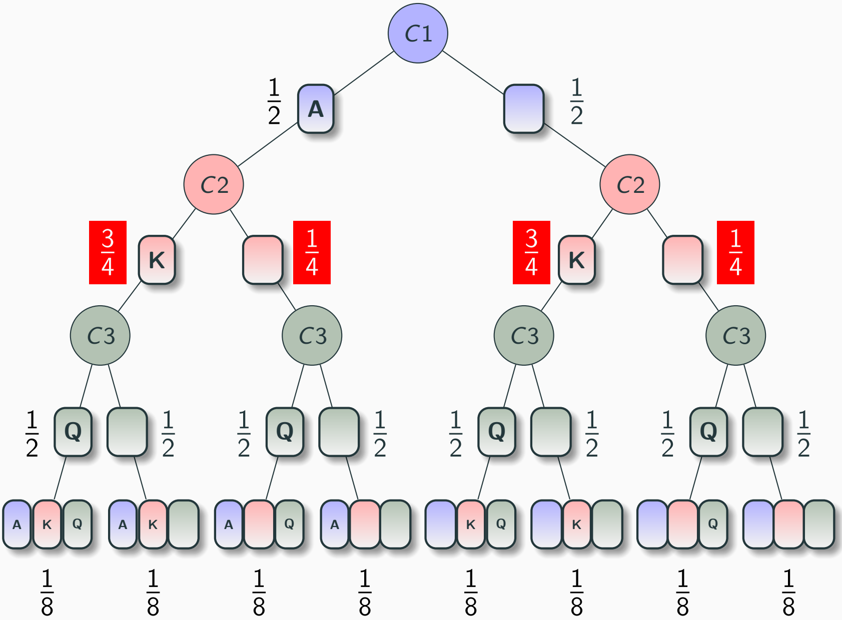
- When dealing the cards, he is 3-times more likely to deal cards so that the blue and red cards “match” (either both contain text or both are blank)

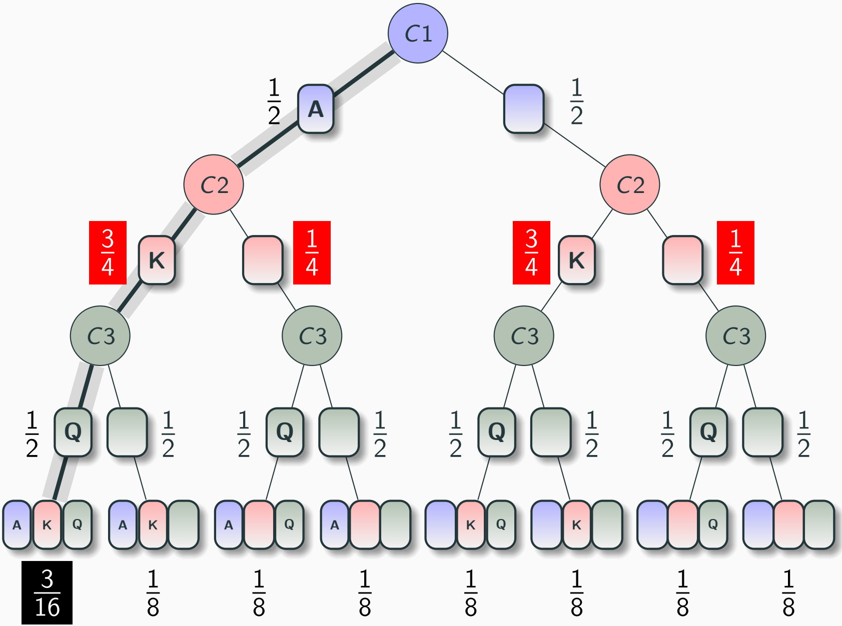


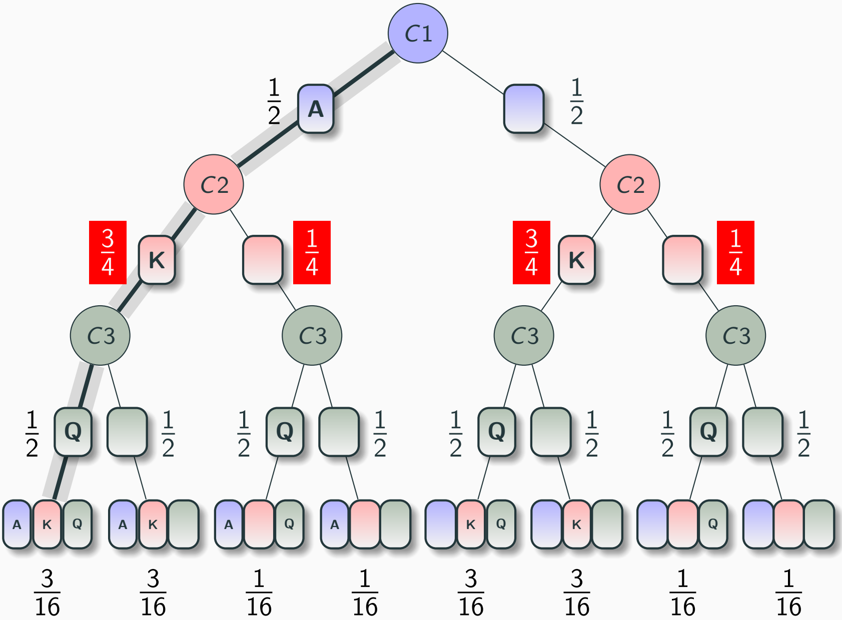
Suppose that the dealer is 3-times more likely to select red K than a blank red card.



C1
1/2
A
1/2
C2
1/2
K
1/2
C2
1/2
K
1/2
C3
C3
C3
C3
1/2
Q
1/2
1/2
Q
1/2
1/2
Q
1/2
1/2
Q
1/2
A K Q
A K
A Q
A
K Q
K
Q
1/8
1/8
1/8
1/8
1/8
1/8
1/8
1/8


C1
1/2
A
1/2
C2
C2
3/4
K
1/4
3/4
K
1/4
C3
C3
C3
C3
1/2
Q
1/2
1/2
Q
1/2
1/2
Q
1/2
1/2
Q
1/2
A K Q
A K
A Q
A
K Q
K
Q
1/8
1/8
1/8
1/8
1/8
1/8
1/8
1/8


C1
1/2
A
1/2
C2
C2
3/4
K
1/4
3/4
K
1/4
C3
C3
C3
C3
1/2
Q
1/2
1/2
Q
1/2
1/2
Q
1/2
1/2
Q
1/2
A K Q
A K
A Q
A
K Q
K
Q
3/16
1/8
1/8
1/8
1/8
1/8
1/8
1/8


C1
1/2
A
1/2
C2
C2
3/4
K
1/4
3/4
K
1/4
C3
C3
C3
C3
1/2
Q
1/2
1/2
Q
1/2
1/2
Q
1/2
1/2
Q
1/2
A K Q
A K
A Q
A
K Q
K
Q
3/16
3/16
1/16
1/16
3/16
3/16
1/16
1/16

prob	A	K	Q	
$\frac{3}{16}$	T	T	T	
$\frac{3}{16}$	T	T	F	
$\frac{1}{16}$	T	F	T	
$\frac{1}{16}$	T	F	F	
$\frac{3}{16}$	F	T	T	
$\frac{3}{16}$	F	T	F	
$\frac{1}{16}$	F	F	T	
$\frac{1}{16}$	F	F	F	

$$Pr(A) =$$

$$Pr(K) =$$

$$Pr(A \wedge K) =$$

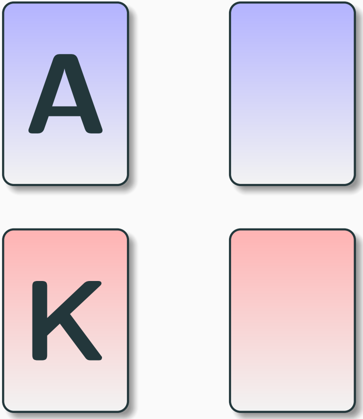
$$Pr(K \to A) =$$

$$Pr(A \mid K) =$$

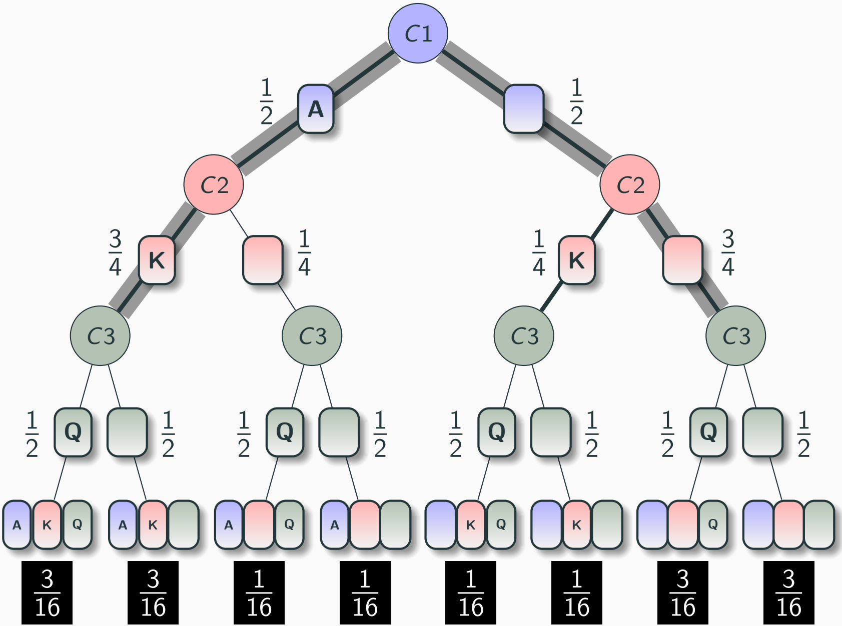
$$Pr(K \mid A) =$$

prob	A	K	Q	
$\frac{3}{16}$	T	T	T	
$\frac{3}{16}$	T	T	F	
$\frac{1}{16}$	T	F	T	
$\frac{1}{16}$	T	F	F	
$\frac{3}{16}$	F	T	T	
$\frac{3}{16}$	F	T	F	
$\frac{1}{16}$	F	F	T	
$\frac{1}{16}$	F	F	F	

$$
\begin{aligned}
Pr(A) &= \tfrac{8}{16} = \tfrac{1}{2} \\
Pr(K) &= \tfrac{12}{16} = \tfrac{3}{4} \\
Pr(A \wedge K) &= \tfrac{6}{16} \\
Pr(K \rightarrow A) &= \tfrac{10}{16} \\
Pr(A \mid K) &= \tfrac{Pr(A \wedge K)}{Pr(K)} = \tfrac{\frac{6}{16}}{\frac{12}{16}} = \tfrac{1}{2} \\
Pr(K \mid A) &= \tfrac{Pr(K \wedge A)}{Pr(A)} = \tfrac{\frac{6}{16}}{\frac{8}{16}} = \tfrac{3}{4}
\end{aligned}
$$



Suppose that the dealer is 3-times more likely to deal cards so that the blue and red cards “match” (either both contain text or both are blank).



C1
1/2
A
1/2
C2
C2
3/4
K
1/4
1/4
K
3/4
C3
C3
C3
C3
1/2
Q
1/2
1/2
Q
1/2
1/2
Q
1/2
1/2
Q
1/2
A K Q
A K
A Q
A
K Q
K
Q
3/16
3/16
1/16
1/16
1/16
1/16
3/16
3/16

prob	A	K	Q	
$\frac{3}{16}$	T	T	T	
$\frac{3}{16}$	T	T	F	
$\frac{1}{16}$	T	F	T	
$\frac{1}{16}$	T	F	F	
$\frac{1}{16}$	F	T	T	
$\frac{1}{16}$	F	T	F	
$\frac{3}{16}$	F	F	T	
$\frac{3}{16}$	F	F	F	

$$Pr(A) =$$

$$Pr(K) =$$

$$Pr(A \wedge K) =$$

$$Pr(A \mid K) =$$

$$Pr(K \mid A) =$$

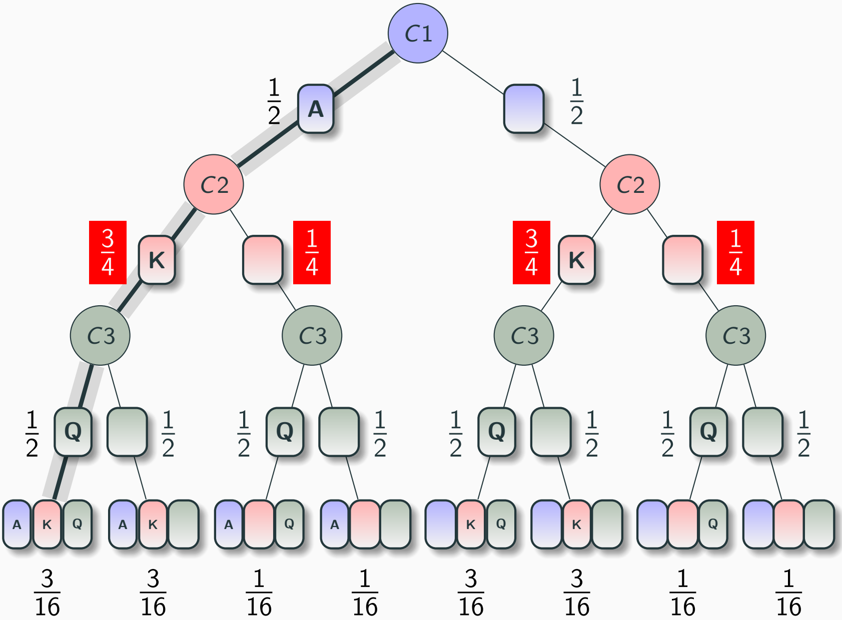
$$Pr(Q \mid A) =$$

prob	A	K	Q	
$\frac{3}{16}$	T	T	T	
$\frac{3}{16}$	T	T	F	
$\frac{1}{16}$	T	F	T	
$\frac{1}{16}$	T	F	F	
$\frac{1}{16}$	F	T	T	
$\frac{1}{16}$	F	T	F	
$\frac{3}{16}$	F	F	T	
$\frac{3}{16}$	F	F	F	

$$
\begin{aligned}
Pr(A) &= \frac{8}{16} = \frac{1}{2} \\
Pr(K) &= \frac{8}{16} = \frac{1}{2} \\
Pr(A \wedge K) &= \frac{6}{16} = \frac{3}{8} \\
Pr(A \mid K) &= \frac{Pr(A \wedge K)}{Pr(K)} = \frac{\frac{6}{16}}{\frac{8}{16}} = \frac{3}{4} \\
Pr(K \mid A) &= \frac{Pr(K \wedge A)}{Pr(A)} = \frac{\frac{6}{16}}{\frac{8}{16}} = \frac{3}{4} \\
Pr(Q \mid A) &= \frac{Pr(Q \wedge A)}{Pr(A)} = \frac{\frac{4}{16}}{\frac{8}{16}} = \frac{1}{2}
\end{aligned}
$$

Biased dealer

- When selecting a red card, he is 3-times more likely to deal a K than a blank.



C1
1/2
A
1/2
C2
C2
3/4
K
1/4
3/4
K
1/4
C3
C3
C3
C3
1/2
Q
1/2
1/2
Q
1/2
1/2
Q
1/2
1/2
Q
1/2
A K Q
A K
A Q
A
K Q
K
Q
3/16
3/16
1/16
1/16
3/16
3/16
1/16
1/16

prob	A	K	Q	
$\frac{3}{16}$	T	T	T	
$\frac{3}{16}$	T	T	F	
$\frac{1}{16}$	T	F	T	
$\frac{1}{16}$	T	F	F	
$\frac{3}{16}$	F	T	T	
$\frac{3}{16}$	F	T	F	
$\frac{1}{16}$	F	F	T	
$\frac{1}{16}$	F	F	F	

$$Pr(A) =$$

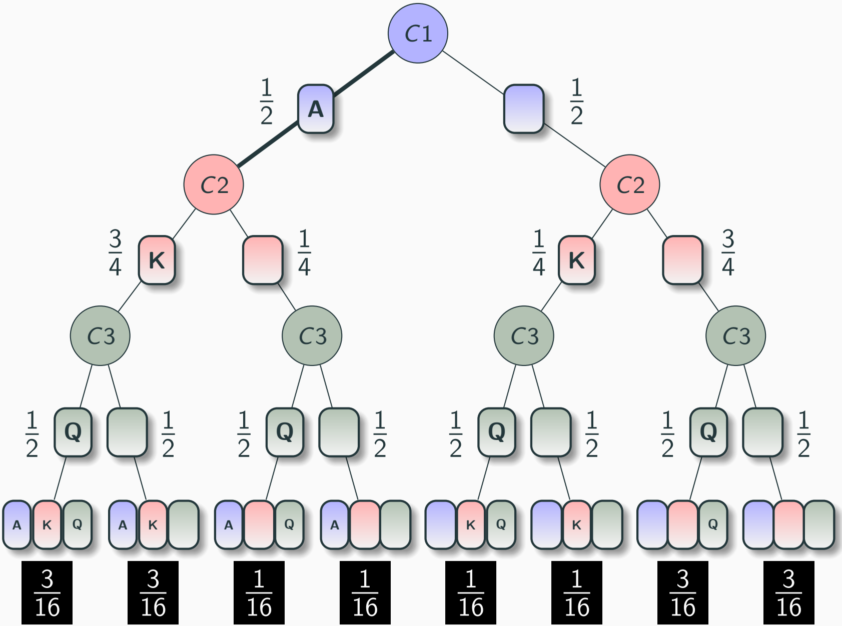
$$Pr(K) =$$

$$Pr(A \wedge K) =$$

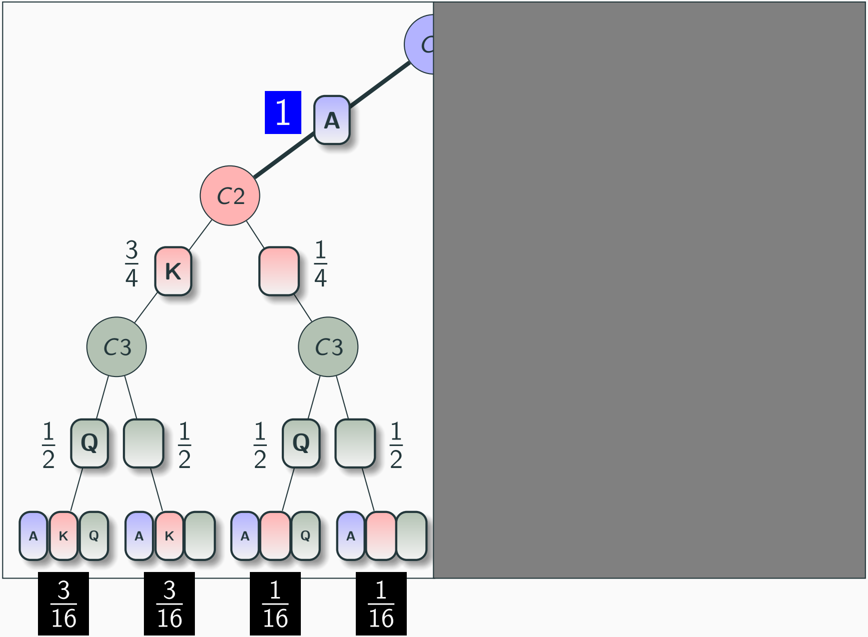
$$Pr(A \mid K) =$$

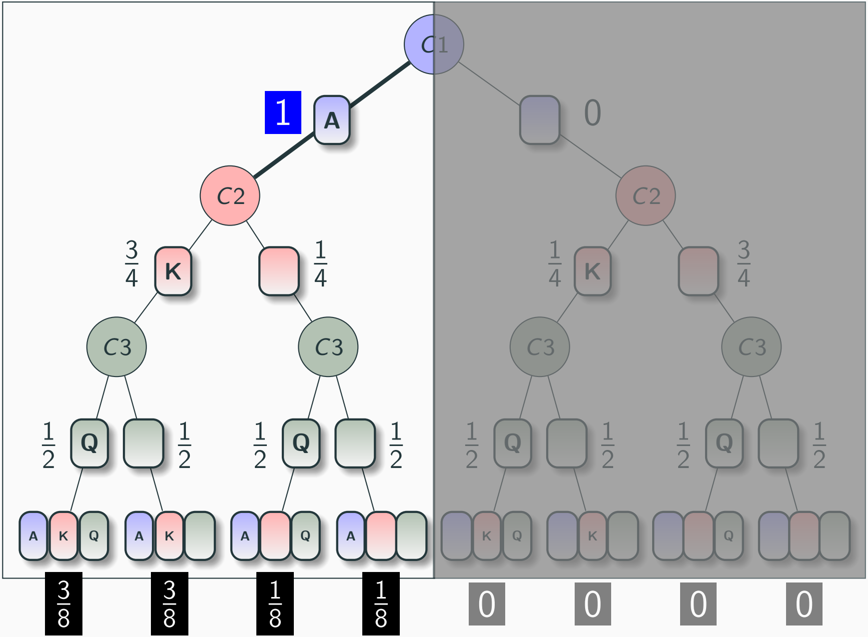
$$Pr(K \mid A) =$$

$$Pr(\cdot \mid A)$$



C1
A
1/2
1/2
C2
C2
K
3/4
1/4
K
1/4
3/4
C3
C3
C3
C3
Q
1/2
1/2
Q
1/2
1/2
Q
1/2
1/2
Q
1/2
1/2
A K Q
A K
A Q
A
K Q
K
Q
3/16
3/16
1/16
1/16
1/16
1/16
3/16
3/16


1
A
C2
3/4
K
1/4
C3
C3
1/2
Q
1/2
1/2
Q
1/2
A K Q
A K
A Q
A
3/16
3/16
1/16
1/16


C1
1
A
0
C2
C2
3/4
K
1/4
1/4
K
3/4
C3
C3
C3
C3
1/2
Q
1/2
1/2
Q
1/2
1/2
Q
1/2
1/2
Q
1/2
A K Q
A K
A Q
A
K Q
K
Q
3/8
3/8
1/8
1/8
0
0
0
0

Kolmogrov Axioms

Given any stochastic truth table, for any formulas φ and ψ:

- $Pr(\varphi) \geq 0$
- If φ is a tautology, then $Pr(\varphi) = 1$
- If φ and ψ are mutually exclusive, then $Pr(\varphi \vee \psi) = Pr(\varphi) + Pr(\psi)$

Laws of Probability

In any stochastic truth table, for all φ, $Pr(\neg\varphi) = 1 - Pr(\varphi)$

In any stochastic truth table, for all φ, if φ is a contradiction, then $Pr(\varphi) = 0$

In any stochastic truth table, for all φ and ψ, if $\varphi \leftrightarrow \psi$ is a tautology (i.e., φ and ψ are tautologically equivalent), then $Pr(\varphi) = Pr(\psi)$

Laws of Probability

In any stochastic truth table, for all φ and ψ,
$Pr(\varphi) = Pr(\varphi \wedge \psi) + Pr(\varphi \wedge \neg\psi)$

In any stochastic truth table, for all φ and ψ,
$Pr(\varphi \vee \psi) = Pr(\varphi) + Pr(\psi) - Pr(\varphi \wedge \psi)$

In any stochastic truth table, for all φ and ψ,
if $\varphi \rightarrow \psi$ is a tautology, then $Pr(\varphi) \leq Pr(\psi)$

Laws of Total Probability

In any stochastic truth table, for all φ and ψ,

$$Pr(\varphi) = Pr(\psi)Pr(\varphi \mid \psi) + Pr(\neg\psi)Pr(\varphi \mid \neg\psi)$$