Reasoning for Humans: Clear Thinking in an Uncertain World

PHIL 171

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Valid Arguments

All Philosophy majors at UMD are required to take logic. Ann is a Philosophy major at UMD. So, Ann is required to take logic.

$$\forall x(M(x) \to L(x)), M(a) \models L(a).$$

Valid Arguments

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- All Philosophy majors at UMD are millionaires. Ann is a Philosophy major at UMD. So, Ann is a millionaire.
- All Democrats voted for Biden. Ann is a Democrat. So, Ann voted for Biden.
- All ravens are black. Tweety is a raven. So, Tweety is black.
- All prime number greater than 2 are odd. 7 is a prime number greater than 2. So, 7 is odd.

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- Is $\forall x(M(x) \to L(x)), \forall x M(x) \Rightarrow \forall x L(x)$ valid?

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- Is $\forall x(M(x) \rightarrow L(x)), \forall x M(x) \Rightarrow \forall x L(x)$ valid? Yes.

- Is $\forall x(M(x) \rightarrow L(x)), \exists x M(x) \Rightarrow L(a)$ valid?

Valid Arguments

- Is $\forall x(M(x) \rightarrow L(x)), \forall x M(x) \Rightarrow \forall x L(x)$ valid? Yes.

- Is $\forall x(M(x) \rightarrow L(x)), \exists x M(x) \Rightarrow L(a)$ valid? No.

Valid Arguments

$$\forall x(M(x) \rightarrow L(x)), \exists x M(x) \not\models L(a)$$

There is a way of interpreting the predicates M and L and the individual constant a such that $\forall x(M(x) \rightarrow L(x))$ and $\exists x M(x)$ are both true, but $L(a)$ is false.

Valid Arguments

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There is a way of interpreting the predicates M and L and the individual constant a such that $\forall x(M(x) \to L(x))$ and $\exists x M(x)$ are both true, but $L(a)$ is false.

For example, consider the set of all integers $\mathbb{N} = \{0, 1, 2, 3, 4, \ldots\}$.

- $M(\cdot)$ means "... is an even integer"
- $L(\cdot)$ means "...is divisible by 2."
- a is the number 3.

Models

The analogue of the truth-value assignment for First Order Logic is a **model**. There are two components of a model:

1. A collection of objects or items called the **domain**.
2. An interpretation that associates with each constant an element of the domain and with each predicate a collection of elements from the domain.

Example

	a	b	M	L
1	✓		$-$	$-$
2			$+$	$+$
3			$+$	$+$
4		✓	$+$	$+$
5			$-$	$+$

- $M(a)$ is false.
- $M(b)$ is true
- $L(a)$ is false

Example

	a	b	M	L
1	✓		$-$	$-$
2			$+$	$+$
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4		✓	$+$	$+$
5			$-$	$+$

- $M(a)$ is false.
- $M(b)$ is true
- $L(a)$ is false
- $\exists x M(x)$ is true
- $\exists x \neg M(x)$ is true

Example

	a	b	M	L
1	✓		$-$	$-$
2			$+$	$+$
3			$+$	$+$
4		✓	$+$	$+$
5			$-$	$+$

- $M(a)$ is false.
- $M(b)$ is true
- $L(a)$ is false
- $\exists x M(x)$ is true
- $\exists x \neg M(x)$ is true
- $\exists y(\neg M(y) \wedge \neg L(y))$ is true

Example

	a	b	M	L
1	✓		$-$	$-$
2			$+$	$+$
3			$+$	$+$
4		✓	$+$	$+$
5			$-$	$+$

- $M(a)$ is false.
- $M(b)$ is true
- $L(a)$ is false
- $\exists x M(x)$ is true
- $\exists x \neg M(x)$ is true
- $\exists y(\neg M(y) \wedge \neg L(y))$ is true
- $\forall x L(x)$ is false
- $\forall x(M(x) \vee L(x))$ is false
- $\forall x(M(x) \rightarrow L(x))$ is true

Example

	a	b	M	L
1	✓		$-$	$-$
2			$+$	$+$
3			$+$	$+$
4		✓	$+$	$+$
5			$-$	$+$

$$\forall x(M(x) \to L(x)), \exists x M(x) \not\models L(a)$$

- $\forall x(M(x) \to L(x))$ is true
- $\exists x M(x)$ is true
- $L(a)$ is false

1. Can you find a model in which $(P(a) \wedge \neg(\exists x\, P(x))$ is true?
   Can you find a model in which $(P(a) \wedge \neg(\exists x\, P(x))$ is false?

1. Can you find a model in which $(P(a) \wedge \neg(\exists x\, P(x))$ is true? No
   Can you find a model in which $(P(a) \wedge \neg(\exists x\, P(x))$ is false? Yes

2. Can you find a model in which $(P(a) \wedge \forall x\, \neg P(x))$ is true?
   Can you find a model in which $(P(a) \wedge \forall x\, \neg P(x))$ is false?

1. Can you find a model in which $(P(a) \wedge \neg(\exists x\, P(x))$ is true? No
   Can you find a model in which $(P(a) \wedge \neg(\exists x\, P(x))$ is false? Yes

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3. Can you find a model in which $(P(a) \wedge \exists x\, \neg P(x))$ is true?
   Can you find a model in which $(P(a) \wedge \exists x\, \neg P(x))$ is false?

1. Can you find a model in which $(P(a) \land \neg(\exists x\, P(x))$ is true? No
   Can you find a model in which $(P(a) \land \neg(\exists x\, P(x))$ is false? Yes

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1. Can you find a model in which $(\forall x\, P(x) \vee \neg\forall x\, P(x))$ is true? Yes
   Can you find a model in which $(\forall x\, P(x) \vee \neg\forall x\, P(x))$ is false? No

2. Can you find a model in which $\forall x\, (P(x) \vee \neg P(x))$ is true?
   Can you find a model in which $\forall x\, (P(x) \vee \neg P(x))$ is false?

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$$\exists x\, P(x) \wedge \exists x\, Q(x) \Rightarrow \exists x\, (P(x) \wedge Q(x))$$

$$\forall x\, P(x) \wedge \forall x\, Q(x) \Rightarrow \forall x\, (P(x) \wedge Q(x))$$

$$\exists x\, (P(x) \vee Q(x)) \Rightarrow \exists x\, P(x) \vee \exists x\, Q(x)$$

$$\forall x\, (P(x) \vee Q(x)) \Rightarrow \forall x\, P(x) \vee \forall x\, Q(x)$$

$$\exists x\, P(x) \wedge \exists x\, Q(x) \not\models \exists x\, (P(x) \wedge Q(x))$$

$$\forall x\, P(x) \wedge \forall x\, Q(x) \models \forall x\, (P(x) \wedge Q(x))$$

$$\exists x\, (P(x) \vee Q(x)) \models \exists x\, P(x) \vee \exists x\, Q(x)$$

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