Reasoning for Humans: Clear Thinking in an Uncertain World

PHIL 171

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Classical Logic "Parameters"

1. *Syntax*: if φ, ψ are sentences, then so are $\neg\varphi$, $\varphi \wedge \psi$, $\varphi \vee \psi$, and $\varphi \to \psi$
2. *Semantics* (truth-functionality): the truth-value of a sentence is a function of the truth-values of its components only
3. *Semantics* (bivalence): sentences are either true or false, with nothing in-between
4. *Consequence*: $\varphi_1 \dots \varphi_n \Rightarrow \psi$ is valid iff ψ is true in every truth value assignment that makes all of $\varphi_1 \dots \varphi_n$ true.

Monotonicity

Monotonicity of Valid Inferences: For all formulas φ, ψ and χ, if $\varphi \models \psi$, then $\varphi, \psi \models \chi$.

Beyond Propositional Logic

All Philosophy majors at UMD are required to take a logic course. Ann is a Philosophy major at UMD. So, Ann is required to take a logic course.

All ravens are black. Tweety is a raven. So, Tweety is black.

Ann brought here laptop to first 20 lectures. So, Ann will bring her laptop to today's lecture.

Beyond Propositional Logic - Quantifiers

We use capital letters $P, Q, R, \ldots$ for variables and lowercase letters $a, b, c, \ldots$ for names. For all variables X and Y and name x, the formulas of syllogistic logic have one of the following forms:

- All X are Y
- Some X are Y
- No X are Y
- Some X are not Y
- x is a Y

All P are Q, a is a $P \models a$ is a Q.

Probability/Inductive Logic

When we *evaluate* arguments, we are interested in two things:

1. Are the premises true?
2. *Supposing* that the premises true, what sort of support do give to the conclusion?

Probability/Inductive Logic

An argument is **deductively valid** if and only if it is *impossible* that its conclusion is false while its premises are true.

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Example

1,100 people, who are known (by some definitive means) to either be infected or not infected by the coronavirus, were given an inexpensive test to see how well it works.

I means the person is infected

P means the person tests positive

	I	P
	T	T
	T	F
	F	T
	F	F

Suppose the 1,100 people were drawn at random from the general population. Now you take the test and test positive. How confident are you that you're infected?

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	I	P
80	T	T
20	T	F
100	F	T
900	F	F

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Weighted Truth Table

A **weighted truth table** is a truth table where each row is assigned a nonnegative number (an integer greater than or equal to 0).

For a formula of propositional logic φ, let $\#(\varphi)$ be the number of ways that φ is true. I.e., it is the sum of the numbers assigned to the rows that make φ true.

Example

	I	P
80	T	T
20	T	F
100	F	T
900	F	F

$$\begin{aligned}
\#(P \wedge I) &= \\
\#(P) &= \\
\#(I) &= \\
\#(\neg P) &= \\
\#(P \vee I) &= \\
\#(P \vee \neg P) &=
\end{aligned}$$

Example

	I	P
80	T	T
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\end{aligned}
$$

Example

	I	P
80	T	T
20	T	F
100	F	T
900	F	F

$$
\begin{aligned}
\#(P \wedge I) &= 80 \\
\#(P) &= 80 + 100 = 180 \\
\#(I) &= 80 + 20 = 100 \\
\#(\neg P) &= 20 + 900 = 920 \\
\#(P \vee I) &= 80 + 20 + 100 = 200 \\
\#(P \vee \neg P) &= 80 + 20 + 100 + 900 = 1100
\end{aligned}
$$

Example

	I	P
0	T	T
100	T	F
180	F	T
820	F	F

$$\begin{aligned}
\#(P \wedge I) &= 0 \\
\#(P) &= 0 + 180 = 180 \\
\#(I) &= 0 + 100 = 100 \\
\#(\neg P) &= 100 + 820 = 920 \\
\#(P \vee I) &= 0 + 100 + 180 = 280 \\
\#(P \vee \neg P) &= 0 + 100 + 180 + 820 = 1100
\end{aligned}$$

Let $\top$ be a special formula that is always true. Then, in any weighted truth-table, $\#(\top)$ is the sum of the numbers assigned to the rows.

In any weighted truth table, for any formula φ,

$$\#(\neg\varphi) = \#(\top) - \#(\varphi)$$