Reasoning for Humans: Clear Thinking in an Uncertain World

PHIL 171

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Beyond Propositional Logic

All Philosophy majors at UMD are required to take a logic course. Ann is a Philosophy major at UMD. So, Ann is required to take a logic course.

Beyond Propositional Logic

All Philosophy majors at UMD are required to take a logic course. Ann is a Philosophy major at UMD. So, Ann is required to take a logic course.

All ravens are black. Tweety is a raven. So, Tweety is black.

All Philosophy majors at UMD are required to take a logic course.

Ann is a Philosophy major at UMD.

So, Ann is required to take a logic course.

H

M

$\therefore S$

(U) All Philosophy majors at UMD are required to take a logic course.

(P) Ann is a Philosophy major at UMD.

So, (L) Ann is required to take a logic course.

$$U, P \Rightarrow L$$

(U) All Philosophy majors at UMD are required to take a logic course.

(P) Ann is a Philosophy major at UMD.

[($U \wedge P \rightarrow L$)
If all Philosophy majors at UMD are required to take a logic course
and Ann is a Philosophy major at UMD,
then Ann is required to take a logic course.]

So, (L) Ann is required to take a logic course.

$$U, P, (U \wedge P) \rightarrow L \Rightarrow L$$

All Philosophy majors at UMD are required to take a logic course.

Ann is a Philosophy major at UMD.

So, Ann is required to take a logic course.

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Ann is a Philosophy major at UMD.

So, Ann is required to take a logic course.

All Philosophy majors at UMD are required to take a logic course.

Bob is a Philosophy major at UMD.

So, Bob is required to take a logic course.

All Philosophy majors at UMD are required to take a logic course.

Carla is a Philosophy major at UMD.

So, Carla is required to take a logic course.

⋮

All Philosophy majors at UMD are required to take a logic course.

Ann is a Philosophy major at UMD.

So, Ann is required to take a logic course.

1. The second premise and the conclusion are represented by different atomic propositions; however both of these statement are about the same single individual ("Ann").
2. The logical form of the first premise involve the quantifier "All".

First-steps in First Order Logic (also called Predicate Logic)

Predicates and Terms

Sentences about John:

- John laughed.
- John is talkative.
- John is in France.
- John likes Mary.
- John is frequently discussed in logic texts.

Predicates and Terms

Sentences about John:

- John laughed. $L(j)$
- John is talkative. $T(j)$
- John is in France. $F(j)$
- John likes Mary. $M(j)$
- John is frequently discussed in logic texts. $D(j)$

Digression: Binary Predicates

- John likes Mary.

Digression: Binary Predicates

- John likes Mary. $L(\quad,\quad)$

Digression: Binary Predicates

- John likes Mary. $L(j, m)$

Atomic Propositions for First Order Logic

Capital letters from the beginning and middle of the alphabet $(A, B, \ldots, P, Q, R, \ldots, U, V, W)$ will be used as **predicates**.

Lower case letters from the beginning and middle of the alphabet $(a, b, \ldots, s, t, u)$ will be used as subjects and will be called **constants**.

An atomic proposition in first order logic is a predicate followed by a constant in parentheses. For example, $A(a)$, $A(b)$, $B(a)$, $\ldots$ are examples of atomic propositions.

Quantifiers, I

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Paraphrase: For any thing, if that thing is a Philosophy major at UMD, then that thing is required to take a logic course.

Quantifiers, I

All Philosophy majors at UMD are required to take a logic course.

Paraphrase: For any thing, if that thing is a Philosophy major at UMD, then that thing is required to take a logic course.

Note: It is important that the symbol that we use to represent the thing that is a Philosophy major at UMD is the same symbol used to represent the thing that is required to take a logic course. A **first order variable** will be represented by a lowercase letter from the end of the alphabet (w, v, x, y, z), possibly with subscripts (e.g., x_1, x_2, $\ldots$).

Quantifiers, II

A **quantifier** is a symbol $\forall$ ("for all") or $\exists$ ("exists") followed by a variable.

$\forall x$, $\forall y$, $\forall z$, ... are called **universal quantifiers**.

$\exists x$, $\exists y$, $\exists z$, ... are called **existential quantifiers**.

Quantifiers, II

A **quantifier** is a symbol $\forall$ ("for all") or $\exists$ ("exists") followed by a variable.

$\forall x$, $\forall y$, $\forall z$, ... are called **universal quantifiers**.

$\exists x$, $\exists y$, $\exists z$, ... are called **existential quantifiers**.

- $\forall x A(x)$ means "everything is an A".
- $\exists x A(x)$ means "something is an A".

Quantifiers

All Philosophy majors at UMD are required to take a logic course.

Quantifiers

All Philosophy majors at UMD are required to take a logic course.

Paraphrase: For any thing, if that thing is a Philosophy major at UMD, then that thing is required to take a logic course.

$U(\cdot)$ means "...is a Philosophy major at UMD."

$L(\cdot)$ means "...is required to take a logic course."

$$\forall x(U(x) \to L(x))$$

Formulas of First Order Logic

1. An atomic proposition of First Order Logic is a predicate applied to a term. For example,

$$A(a), A(b), \ldots A(x), A(y), \ldots$$

   are atomic proposition of First Order Logic. Each atomic propositions of First Order Logic is a formula of First Order Logic.
2. If X is a formula for First Order Logic, then so is $\neg X$.
3. If X and Y are formulas of First Order Logic, then so are

$$(X \vee Y), (X \wedge Y), (X \rightarrow Y), \text{ and } (X \leftrightarrow Y).$$

4. If X is a formula of First Order Logic, then so is a quantifier appended to X, i.e., $\forall x X$ and $\exists x X$ are formulas of First Order Logic.
5. Nothing else is a formula.

Examples

1. $P(a)$
2. $(P(a) \wedge Q(a))$
3. $(P(x) \wedge \neg Q(b))$
4. $\forall x P(x)$
5. $\forall x(P(x) \rightarrow Q(x))$
6. $\exists y(P(x) \wedge \neg Q(y))$
7. $(Q(a) \rightarrow \forall y(P(y) \wedge \neg R(a)))$
8. $(\neg \forall x \neg P(x) \rightarrow \exists x P(x))$

- Maggie is a dog, and she is black.

- Snowball is a cat, and she is white.

- Zebbie is a zebra, and he is black and white.

- Maggie is a dog, and she is black.
  $(D(m) \wedge B(m))$

- Snowball is a cat, and she is white.

- Zebbie is a zebra, and he is black and white.

- Maggie is a dog, and she is black.
  $(D(m) \wedge B(m))$

- Snowball is a cat, and she is white.
  $(C(s) \wedge W(s))$

- Zebbie is a zebra, and he is black and white.

- Maggie is a dog, and she is black.
  $(D(m) \wedge B(m))$

- Snowball is a cat, and she is white.
  $(C(s) \wedge W(s))$

- Zebbie is a zebra, and he is black and white.
  $(Z(z) \wedge (B(z) \wedge W(z))$

- Maggie is a dog, and she is black.
  $(D(m) \wedge B(m))$

- Snowball is a cat, and she is white.
  $(C(s) \wedge W(s))$

- Zebbie is a zebra, and he is black and white.
  $(Z(z) \wedge P(z))$

Translations

$H(x)$	x is happy.
$S(x)$	x is a student.

All students are happy.

$$(\forall x)(S(x) \to H(x))$$

$$(\forall x)(S(x) \land H(x))$$

Some students are happy.

$$(\exists x)(S(x) \to H(x))$$

$$(\exists x)(S(x) \land H(x))$$

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All students are happy.

✓ $(\forall x)(S(x) \to H(x))$

✗ $(\forall x)(S(x) \land H(x))$ (Everyone is a student and happy.)

Some students are happy.

$(\exists x)(S(x) \to H(x))$

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Translations

$H(x)$	x is happy.
$S(x)$	x is a student.

All students are happy.

- ✓ $(\forall x)(S(x) \rightarrow H(x))$
- ✗ $(\forall x)(S(x) \land H(x))$ (Everyone is a student and happy.)

Some students are happy.

- ✗ $(\exists x)(S(x) \rightarrow H(x))$ (There is someone that is either not a student or happy.)
- ✓ $(\exists x)(S(x) \land H(x))$

Examples

$L(x)$	x is a logician
$M(x)$	x is a mathematician

All logicians are mathematicians.

Some logicians are mathematicians.

Examples

$L(x)$	x is a logician
$M(x)$	x is a mathematician

All logicians are mathematicians.

$(\forall x)(L(x) \to M(x))$

Some logicians are mathematicians.

$(\exists x)(L(x) \land M(x))$

Examples

No logician is a mathematician.

$L(x)$	x is a logician
$M(x)$	x is a mathematician

Some logicians are not mathematicians.

Every logician is not a mathematician.

Not every logician is a mathematician.

Examples

No logician is a mathematician.

$\neg(\exists x)(L(x) \wedge M(x))$

$L(x)$	x is a logician
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No logician is a mathematician.

$\neg(\exists x)(L(x) \wedge M(x))$

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Some logicians are not mathematicians.

$(\exists x)(L(x) \wedge \neg M(x))$

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$L(x)$	x is a logician
$M(x)$	x is a mathematician

No logician is a mathematician.

$\neg(\exists x)(L(x) \wedge M(x))$

Some logicians are not mathematicians.

$(\exists x)(L(x) \wedge \neg M(x))$

Every logician is not a mathematician.

$(\forall x)(L(x) \to \neg M(x))$

Not every logician is a mathematician.

Examples

No logician is a mathematician.

$\neg(\exists x)(L(x) \wedge M(x))$

$L(x)$	x is a logician
$M(x)$	x is a mathematician

Some logicians are not mathematicians.

$(\exists x)(L(x) \wedge \neg M(x))$

Every logician is not a mathematician.

$(\forall x)(L(x) \rightarrow \neg M(x))$

Not every logician is a mathematician.

$\neg(\forall x)(L(x) \rightarrow M(x))$

every **A** is **B**	$(\forall x)(A(x) \to B(x))$
some **A** is **B**	$(\exists x)(A(x) \wedge B(x))$
no **A** is **B**	$\neg(\exists x)(A(x) \wedge B(x))$
some **A** is not **B**	$(\exists x)(A(x) \wedge \neg B(x))$
every **A** is a non-**B**	$(\forall x)(A(x) \to \neg B(x))$
not every **A** is **B**	$\neg(\forall x)(A(x) \to B(x))$