Reasoning for Humans: Clear Thinking in an Uncertain World

PHIL 171

Eric Pacuit

Department of Philosophy
University of Maryland
pacuit.org

Wason Selection Task

P. C. Wason. *Reasoning about a rule*. Quarterly Journal of Experimental Psychology, 20:273 - 281, 1968.

Wason Selection Task

You are shown a set of four cards placed on a table, each of which has a number on one side and a letter on the other side. Also below is a rule which applies only to the four cards. Your task is to decide which if any of these four cards you *must* turn in order to decide if the rule is true. Don't turn unnecessary cards.

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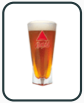
Rule: If there is a vowel on one side, then there is an even number on the other side.

A	K	4	7

Suppose that you're working as a security executive in a bar (you're a bouncer). It's your job to ensure that the rule governing the consumption of alcohol is strictly enforced. It states:

Rule: If a person drinks an alcoholic drink, then they must be over the age of 21 years old.

Please indicate which card or cards you definitely need to turn over, and only that or those cards, in order to determine whether the rule is broken in the case of each of the four customers.



22

17

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A	K	4	7

V	E	$(V \to E)$
T	T	T
T	F	F
F	T	T
F	F	T

P: there is a vowel

Q: there is an even number

Rule: If there is a vowel on one side, then there is an even number on the other side.

A $\quad$ K $\quad$ 4 $\quad$ 7

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A K 4 7

Denying the Antecedent

$V \rightarrow E, \neg V \Rightarrow ??$

Affirming the Consequent

$V \rightarrow E, E \Rightarrow ??$

Modus Ponens

$V \rightarrow E, V \Rightarrow E$

Modus Tollens

$V \rightarrow E, \neg E \Rightarrow \neg V$

Responses

If there is a vowel on one side, then there is an even number on the other side.

	$\boxed{\text{A}}$	$\boxed{\text{K}}$	$\boxed{4}$	$\boxed{7}$
Typical experimental results	89%	16%	62%	25%

Responses

If there is a vowel on one side, then there is an even number on the other side.

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Typical experimental results	89%	16%	62%	25%

If a person is drinking beer, then the person must be over 21.

			22	17
Typical experimental results	95%	0.0025%	0.0025%	80%

H. Mercier and D. Sperber. *The Enigma of Reason*. Harvard University Press, 2019.

K. Stenning and M. van Lambalgen. *Human reasoning and cognitive science*. MIT Press, 2008.

Given an argument in English, is the argument valid?

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$$\Downarrow$$

Translate to the language of propositional logic.

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If yes, then the original argument in English is valid
(assuming that the translation correctly represents the argument).

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Translate to the language of propositional logic.

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Is the translated argument valid?

If yes, then the original argument in English is valid
(assuming that the translation correctly represents the argument).

If no, then the original argument in English may or may not be valid.
(implicit premises might need to be made explicit, other logical systems)

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There might be a mismatch between formulas of propositional logic and corresponding statements in English.

$$X \vee Y, \neg Y \models X$$

$$X \vee Y \not\models X$$

$X \vee Y, \neg Y \models X$

You will get an A or B in PHIL 171. You will not get a B in PHIL 171. Therefore, you will get an A in PHIL 171.

$X \vee Y \not\models X$

You will get an A or B in PHIL 171. Therefore, you will get an A in PHIL 171.

Conditionals

1. If it's a square, then it's a rectangle.
2. If $x = 5$, then $x + 3 = 8$.
3. If you strike the match, then it will light.
4. If you had struck the match, then it would have lit.

Conditionals play a prominent role in mathematical, practical and causal reasoning.

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1. If Hillary Clinton had won the election in 2016, then she would be the 45th president of the USA.
2. If Hillary Clinton had won the election in 2016, then UMD would build a dorm on the moon.

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2. If Hillary Clinton had won the election in 2016, then UMD would build a dorm on the moon.

According to the semantics of sentential logic, both 1. and 2. are true (because the antecedent if false). However, it seems that 1. is true while 2. is false.

- If I weighed more than 300 pounds, I would weigh more than 200 pounds.
- If I weighed more than 300 pounds, I would weigh less than 10 pounds.

$\neg X \models X \to Y \qquad\qquad X \models \neg X \to Y$

$\neg X \models X \to Y$ $\qquad$ $X \models \neg X \to Y$

College Park is in Maryland. ?? So, if College Park is not in Maryland, then Obama is a Republican.

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$Y \models X \to Y$

$\neg X \models X \rightarrow Y$ $\qquad$ $X \models \neg X \rightarrow Y$

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$Y \models X \rightarrow Y$

Eric was in College Park this morning. ?? So, if Eric was in NYC this morning, then Eric was in College Park this morning.

$X \to Y \models \neg Y \to \neg X$

$X \to Y \models \neg Y \to \neg X$

If Gödel had lived past 1978, he would not be alive today. ?? So, if Gödel was alive today, then he would not have lived past 1978.

$Y \to Z, X \to Y \models X \to Z$

$Y \to Z, X \to Y \models X \to Z$

If I quit my job, I won't be able to afford my apartment. But if I win 10 million dollars, I will quit my job. ?? So, if I win 10 million dollars, I won't be able to afford my apartment.

Monotonicity, I

$$X \to Y \models (X \land Z) \to Y$$

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If I put sugar in my coffee, then it will taste good. ?? So, if I put sugar and gasoline in my coffee, then it will taste good.

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$X \to Y \models (X \land Z) \to Y$

If I put sugar in my coffee, then it will taste good. ?? So, if I put sugar and gasoline in my coffee, then it will taste good.

If this match is struck, then it will light. ?? So, if this match is struck and soaked overnight, then it will light.