Reasoning for Humans: Clear Thinking in an Uncertain World

PHIL 171

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Boolean Equivalences

DeMorgan's Law	$\neg(\varphi \wedge \psi) \approx (\neg\varphi \vee \neg\psi)$
DeMorgan's Law	$\neg(\varphi \vee \psi) \approx (\neg\varphi \wedge \neg\psi)$
Conditional	$(\varphi \to \psi) \approx (\neg\varphi \vee \psi)$
Distribution	$(\varphi \vee (\psi \wedge \chi)) \approx ((\varphi \vee \psi) \wedge (\varphi \vee \chi))$
Distribution	$(\varphi \wedge (\psi \vee \chi)) \approx ((\varphi \wedge \psi) \vee (\varphi \wedge \chi))$

$$\varphi \approx \varphi \wedge (\psi \vee \neg\psi)$$

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$$\varphi \approx (\varphi \vee \psi) \wedge (\varphi \vee \neg\psi)$$

Valid Inference Rules

Name	Valid inference rule
Modus Ponens	$\varphi, \varphi \to \psi \models \psi$
Modus Tollens	$\varphi \to \psi, \neg\psi \models \neg\varphi$
Disjunctive Syllogism	$\varphi \vee \psi, \neg\varphi \models \psi$
Transitivity	$\varphi \to \psi, \psi \to \chi \models \varphi \to \chi$

Invalid Inferences

Name	Invalid inference rule
Denying the Antecedent	$\neg\varphi, \varphi \rightarrow \psi \not\models \neg\psi$
Affirming the Consequent	$\psi, \varphi \rightarrow \psi \not\models \varphi$
Affirming a Disjunct	$\varphi \vee \psi, \varphi \not\models \neg\psi$

H. Mercier and D. Sperber. *The Enigma of Reason*. Harvard University Press, 2019.

K. Stenning and M. van Lambalgen. *Human reasoning and cognitive science*. MIT Press, 2008.

Given an argument in English, is the argument valid?

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$$\Downarrow$$

Translate to the language of propositional logic.

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(assuming that the translation correctly represents the argument).

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Translate to the language of propositional logic.

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If yes, then the original argument in English is valid
(assuming that the translation correctly represents the argument).

If no, then the original argument in English may or may not be valid.
(implicit premises might need to be made explicit, other logical systems)

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After replacing the atomic propositions with statements in English, is the resulting argument valid?

There might be a mismatch between formulas of propositional logic and corresponding statements in English.

$$\varphi \lor \psi, \neg\psi \models \varphi$$

$$\varphi \lor \psi \not\models \varphi$$

$\varphi \vee \psi, \neg\psi \models \varphi$

You will get an A or B in PHIL 171. You will not get a B in PHIL 171. Therefore, you will get an A in PHIL 171.

$\varphi \vee \psi \not\models \varphi$

You will get an A or B in PHIL 171. Therefore, you will get an A in PHIL 171.

Conditionals

1. If it's a square, then it's a rectangle.
2. If $x = 5$, then $x + 3 = 8$.
3. If you strike the match, then it will light.
4. If you had struck the match, then it would have lit.

Conditionals play a prominent role in mathematical, practical and causal reasoning.

φ	ψ	$(\varphi \to \psi)$
T	T	T
T	F	F
F	T	T
F	F	T

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1. If Hillary Clinton had won the election in 2016, then she would be the 45th president of the USA.

2. If Hillary Clinton had won the election in 2016, then UMD would build a dorm on the moon.

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φ	ψ	$(\varphi \rightarrow \psi)$
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According to the semantics of sentential logic, both 1. and 2. are true (because the antecedent if false). However, it seems that 1. is true while 2. is false.

- If I weighed more than 300 pounds, I would weigh more than 200 pounds.
- If I weighed more than 300 pounds, I would weigh less than 10 pounds.

$\neg\varphi \models \varphi \to \psi \qquad \varphi \models \neg\varphi \to \psi$

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College Park is in Maryland. ?? So, if College Park is not in Maryland, then Obama is a Republican.

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$\psi \models \varphi \to \psi$

Eric was in College Park this morning. ?? So, if Eric was in NYC this morning, then Eric was in College Park this morning.

$\varphi \rightarrow \psi \models \neg\psi \rightarrow \neg\varphi$

$\varphi \to \psi \models \neg\psi \to \neg\varphi$

If Gödel had lived past 1978, he would not be alive today. ?? So, if Gödel was alive today, then he would not have lived past 1978.

$$\psi \to \chi, \varphi \to \psi \models \varphi \to \chi$$

$\psi \to \chi, \varphi \to \psi \models \varphi \to \chi$

If I quit my job, I won't be able to afford my apartment. But if I win 10 million dollars, I will quit my job. ?? So, if I win 10 million dollars, I won't be able to afford my apartment.

Parameters of a Logic

The set of parameters characterizing a logic can be divided in three subsets:

1. Choice of formal language
2. Choice of a semantics for the formal language
3. Choice of a definition of valid arguments, or valid inference rules, in the language

Classical Logic "Parameters"

1. *Syntax*: if φ, ψ are sentences, then so are $\neg\varphi$, $\varphi \wedge \psi$, $\varphi \vee \psi$, and $\varphi \to \psi$
2. *Semantics* (truth-functionality): the truth-value of a sentence is a function of the truth-values of its components only
3. *Semantics* (bivalence): sentences are either true or false, with nothing in-between
4. *Consequence*: $\varphi_1 \ldots \varphi_n \Rightarrow \psi$ is valid iff ψ is true in every truth value assignment that makes all of $\varphi_1 \ldots \varphi_n$ true.

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Domains to which classical logic is applicable must satisfy these four assumptions.

Monotonicity, I

$\varphi \to \psi \models (\varphi \land \chi) \to \psi$

Monotonicity, I

$\varphi \rightarrow \psi \models (\varphi \wedge \chi) \rightarrow \psi$

If I put sugar in my coffee, then it will taste good. ?? So, if I put sugar and gasoline in my coffee, then it will taste good.

Monotonicity, I

$\varphi \rightarrow \psi \models (\varphi \wedge \chi) \rightarrow \psi$

If I put sugar in my coffee, then it will taste good. ?? So, if I put sugar and gasoline in my coffee, then it will taste good.

If this match is struck, then it will light. ?? So, if this match is struck and soaked overnight, then it will light.

Monotonicity, II

Monotonicity of Valid Inferences: For all formulas φ, ψ and χ, if $\varphi \models \psi$, then $\varphi, \psi \models \chi$.

Suppression Task:

- If she has an essay to finish, then she will study late in the library.
- She has an essay to finish.

1. She will study late in the library.
2. She will not study late in the library.
3. She may or may not study late in the library.

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- If she has an essay to finish, then she will study late in the library.
- If the library stays open, then she will study late in the library.
- She has an essay to finish.

Beyond Propositional Logic

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All ravens are black. Tweety is a raven. So, Tweety is black.

Beyond Propositional Logic

We use capital letters $P, Q, R, \ldots$ for variables and lowercase letters $a, b, c, \ldots$ for names. For all variables X and Y and name x, the formulas of syllogistic logic have one of the following forms:

- All X are Y
- Some X are Y
- No X are Y
- Some X are not Y
- x is a Y

All P are Q, a is a $P \models a$ is a Q.

Closed-world reasoning

Negation as failure
Suppose you are interested in whether there are any direct flights from Amsterdam to Cleveland, Ohio.

After searching online at a number of relevant sites (Expedia, Orbitz, KLM, etc.), you do not find any. You conclude that there are *no direct flights between Amsterdam and Cleveland*.

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- Actual human reasoning falls short of prescriptive standards, so there is room for improvement by suitable education
- Reasoning rarely happens in real life: we have developed "fast and frugal algorithms" which allow us to take quick decisions which are optimal given constraints of time and energy.

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- Although formal models are idealizations which abstract away some aspects of natural systems, they are useful idealizations that help us understand many natural relationships and regularities.

- Similarly, studying arguments expressible in formal languages allows us to develop powerful tools for testing validity. We won't be able to capture all valid arguments this way. But, we can grasp many.