Reasoning for Humans: Clear Thinking in an Uncertain World

PHIL 171

Eric Pacuit

Department of Philosophy
University of Maryland
pacuit.org

If an argument has premises that are not satisfiable, then the argument is valid.

If all the premises and the conclusion are true, then the argument is valid.

Choose all the formulas that are tautologically equivalent to P:

P	Q	$P \to P$	$\neg P \to P$	$(P \wedge Q) \vee (P \wedge \neg Q)$	$(P \vee Q) \wedge (P \vee \neg Q)$
T	T	T	T	T	T
T	F	T	T	T	T
F	T	T	F	F	F
F	F	T	F	F	F

$$\varphi \approx \varphi \wedge (\psi \vee \neg\psi)$$

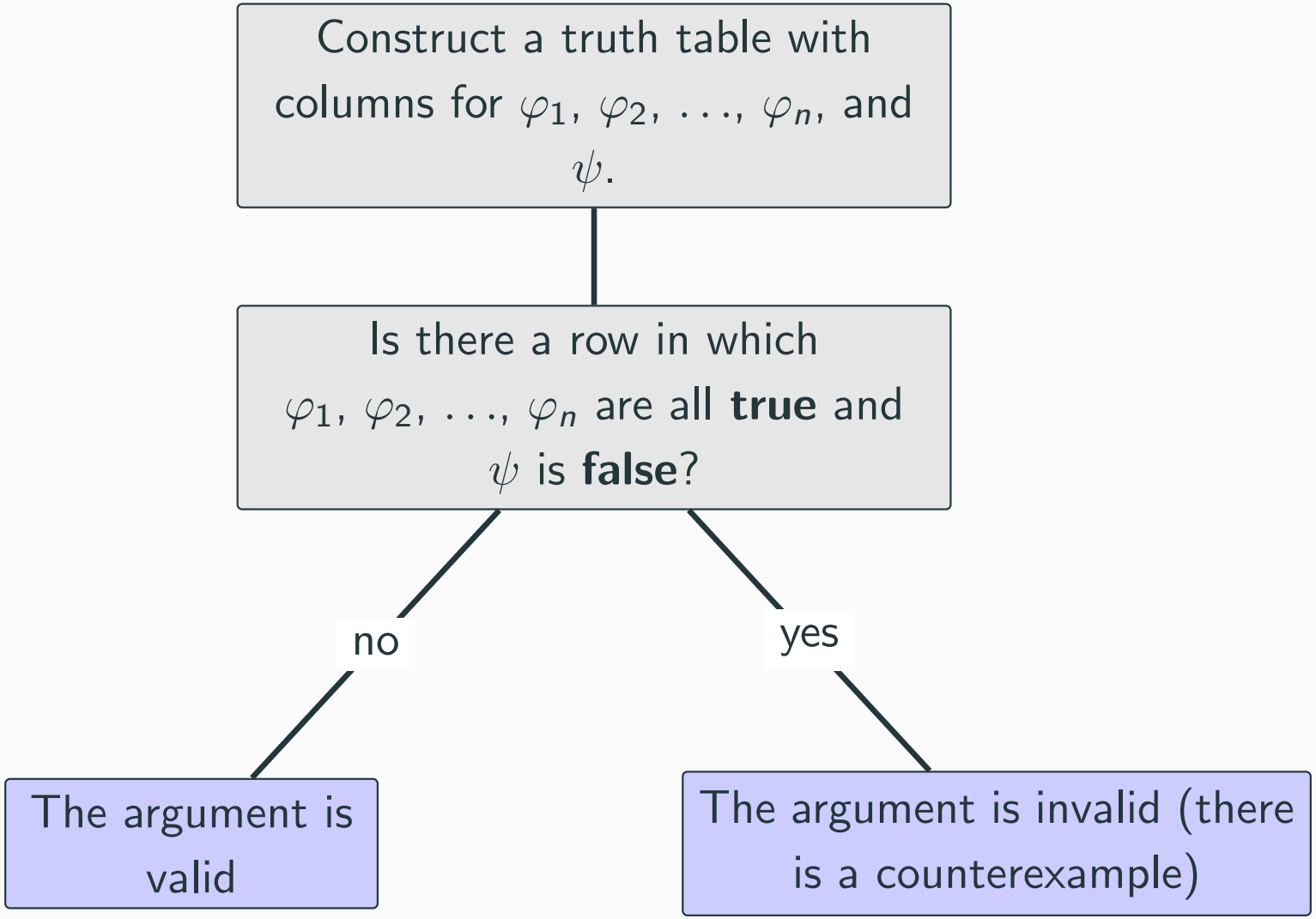
$$\varphi \approx (\varphi \wedge \psi) \vee (\varphi \wedge \neg\psi)$$

$$\varphi \approx \varphi \wedge (\psi \vee \neg\psi)$$

$$\varphi \approx (\varphi \wedge \psi) \vee (\varphi \wedge \neg\psi)$$

$$\varphi \approx \varphi \vee (\psi \wedge \neg\psi)$$

$$\varphi \approx (\varphi \vee \psi) \wedge (\varphi \vee \neg\psi)$$


Construct a truth table with columns for φ_1, φ_2, ..., φ_n, and ψ.
Is there a row in which φ_1, φ_2, ..., φ_n are all **true** and ψ is **false**?
no
yes
The argument is valid
The argument is invalid (there is a counterexample)

Weird Cases: Which arguments are valid?

$$B \Rightarrow (A \vee \neg A)$$

$$A \vee \neg A \Rightarrow B$$

$$A \wedge \neg A \Rightarrow B$$

Weird Cases

- Suppose that an argument contains a tautology as a premise. Is the argument valid?

Weird Cases

- Suppose that an argument contains a tautology as a premise. Is the argument valid? Maybe.

Weird Cases

- Suppose that an argument contains a tautology as a premise. Is the argument valid? Maybe.

- Suppose that the argument contains a contradiction as a premise. Is the argument valid?

Weird Cases

- Suppose that an argument contains a tautology as a premise. Is the argument valid? Maybe.

- Suppose that the argument contains a contradiction as a premise. Is the argument valid? Yes.

Weird Cases

- Suppose that an argument contains a tautology as a premise. Is the argument valid? Maybe.

- Suppose that the argument contains a contradiction as a premise. Is the argument valid? Yes.

- Suppose that the argument contains a tautology as a conclusion. Is the argument valid?

Weird Cases

- Suppose that an argument contains a tautology as a premise. Is the argument valid? Maybe.

- Suppose that the argument contains a contradiction as a premise. Is the argument valid? Yes.

- Suppose that the argument contains a tautology as a conclusion. Is the argument valid? Yes.

Weird Cases

- Suppose that an argument contains a tautology as a premise. Is the argument valid? Maybe.
- Suppose that the argument contains a contradiction as a premise. Is the argument valid? Yes.
- Suppose that the argument contains a tautology as a conclusion. Is the argument valid? Yes.
- Suppose that the argument contains a contradiction as a conclusion. Is the argument valid?

Weird Cases

- Suppose that an argument contains a tautology as a premise. Is the argument valid? Maybe.
- Suppose that the argument contains a contradiction as a premise. Is the argument valid? Yes.
- Suppose that the argument contains a tautology as a conclusion. Is the argument valid? Yes.
- Suppose that the argument contains a contradiction as a conclusion. Is the argument valid? Maybe.

Valid Inference Rules

Name	Valid inference rule
Modus Ponens	$\varphi, \varphi \to \psi \models \psi$
Modus Tollens	$\varphi \to \psi, \neg\psi \models \neg\varphi$
Disjunctive Syllogism	$\varphi \vee \psi, \neg\varphi \models \psi$
Transitivity	$\varphi \to \psi, \psi \to \chi \models \varphi \to \chi$

Invalid Inferences

Name	Invalid inference rule
Denying the Antecedent	$\neg\varphi, \varphi \to \psi \not\models \neg\psi$
Affirming the Consequent	$\psi, \varphi \to \psi \not\models \varphi$
Affirming a Disjunct	$\varphi \vee \psi, \varphi \not\models \neg\psi$

Wason Selection Task

P. C. Wason. *Reasoning about a rule*. Quarterly Journal of Experimental Psychology, 20:273 - 281, 1968.

Wason Selection Task

You are shown a set of four cards placed on a table, each of which has a number on one side and a letter on the other side. Also below is a rule which applies only to the four cards. Your task is to decide which if any of these four cards you *must* turn in order to decide if the rule is true. Don't turn unnecessary cards.

Wason Selection Task

You are shown a set of four cards placed on a table, each of which has a number on one side and a letter on the other side. Also below is a rule which applies only to the four cards. Your task is to decide which if any of these four cards you *must* turn in order to decide if the rule is true. Don't turn unnecessary cards.

Rule: If there is a vowel on one side, then there is an even number on the other side.

Suppose that you're working as a security executive in a bar (you're a bouncer). It's your job to ensure that the rule governing the consumption of alcohol is strictly enforced. It states:

Rule: If a person drinks an alcoholic drink, then they must be over the age of 21 years old.

Please indicate which card or cards you definitely need to turn over, and only that or those cards, in order to determine whether the rule is broken in the case of each of the four customers.

22

17

Rule: If there is a vowel on one side, then there is an even number on the other side.

V	E	$(V \to E)$
T	T	T
T	F	F
F	T	T
F	F	T

P: there is a vowel

Q: there is an even number

Rule: If there is a vowel on one side, then there is an even number on the other side.

V	E	$(V \to E)$
T	T	T
T	F	F
F	T	T
F	F	T

V: there is a vowel

E: there is an even number

Rule: If there is a vowel on one side, then there is an even number on the other side.

V	E	$(V \to E)$
T	T	T
T	F	F
F	T	T
F	F	T

V: there is a vowel

E: there is an even number

Rule: If there is a vowel on one side, then there is an even number on the other side.

V	E	$(V \to E)$
T	T	T
T	F	F
F	T	T
F	F	T

V: there is a vowel

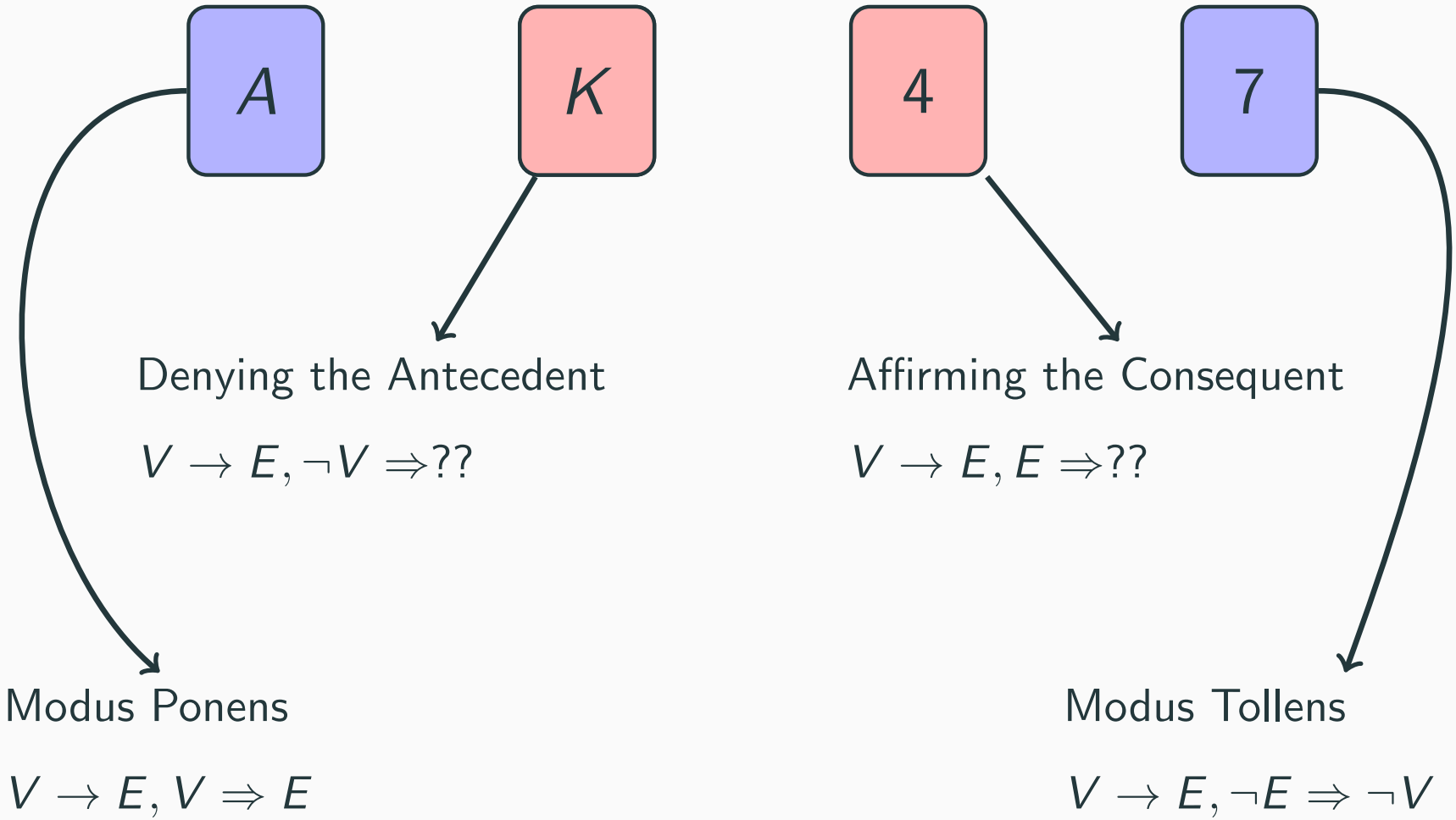
E: there is an even number

Rule: If there is a vowel on one side, then there is an even number on the other side.

V	E	$(V \to E)$
T	T	T
T	F	F
F	T	T
F	F	T

V: there is a vowel

E: there is an even number


A
K
4
7
Denying the Antecedent
$V \to E, \neg V \Rightarrow ??$
Affirming the Consequent
$V \to E, E \Rightarrow ??$
Modus Ponens
$V \to E, V \Rightarrow E$
Modus Tollens
$V \to E, \neg E \Rightarrow \neg V$

Responses

If there is a vowel on one side, then there is an even number on the other side.

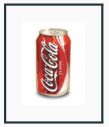
	A	K	4	7
Typical experimental results	89%	16%	62%	25%

Responses

If there is a vowel on one side, then there is an even number on the other side.

	A	K	4	7
Typical experimental results	89%	16%	62%	25%

If a person is drinking beer, then the person must be over 21.

			22	17
Typical experimental results	95%	0.0025%	0.0025%	80%

H. Mercier and D. Sperber. *The Enigma of Reason*. Harvard University Press, 2019.

K. Stenning and M. van Lambalgen. *Human reasoning and cognitive science*. MIT Press, 2008.