Reasoning for Humans: Clear Thinking in an Uncertain World

PHIL 171

Eric Pacuit

Department of Philosophy
University of Maryland
pacuit.org

Recap: Truth Tables

φ	ψ	$(\varphi \wedge \psi)$
T	T	T
T	F	F
F	T	F
F	F	F

φ	ψ	$(\varphi \vee \psi)$
T	T	T
T	F	T
F	T	T
F	F	F

φ	ψ	$(\varphi \rightarrow \psi)$
T	T	T
T	F	F
F	T	T
F	F	T

φ	ψ	$(\varphi \leftrightarrow \psi)$
T	T	T
T	F	F
F	T	F
F	F	T

φ	$\neg\varphi$
T	F
F	T

Valid Argument:

Valid Argument: An argument is valid provided that there is no truth value assignment that makes all the premises true and the conclusion false.

Valid Argument: An argument is valid provided that there is no truth value assignment that makes all the premises true and the conclusion false. (So, any truth-value assignment that makes all the premises true also makes the conclusion true).

Invalid Argument:

Valid Argument: An argument is valid provided that there is no truth value assignment that makes all the premises true and the conclusion false. (So, any truth-value assignment that makes all the premises true also makes the conclusion true).

Invalid Argument: An argument is invalid just in case it is not valid, i.e., if there is some truth-value assignment that makes the premises true and the conclusion false.

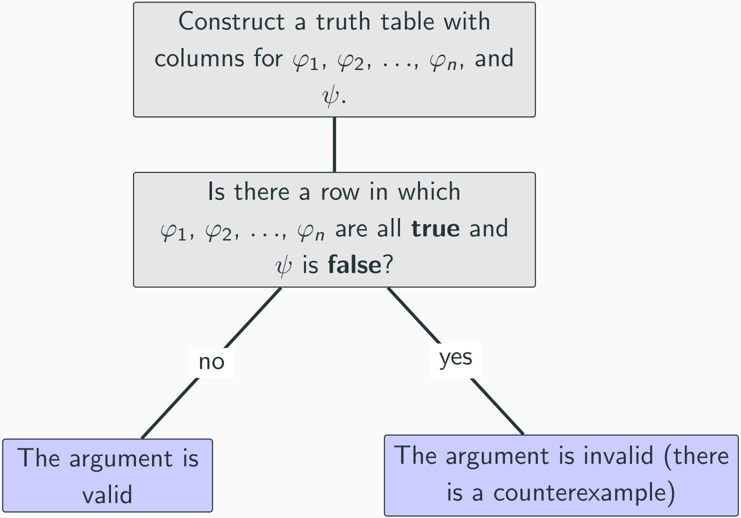
Counterexample: A truth-value assignment that makes the premises of an argument true and its conclusion false is called a counterexample to the argument.

Valid Argument: An argument is valid provided that there is no truth value assignment that makes all the premises true and the conclusion false. (So, any truth-value assignment that makes all the premises true also makes the conclusion true).

Invalid Argument: An argument is invalid just in case it is not valid, i.e., if there is some truth-value assignment that makes the premises true and the conclusion false.

Counterexample: A truth-value assignment that makes the premises of an argument true and its conclusion false is called a counterexample to the argument.

So, an argument if valid if there are no counterexamples.


Construct a truth table with columns for φ_1, φ_2, . . ., φ_n, and ψ.
Is there a row in which φ_1, φ_2, . . ., φ_n are all **true** and ψ is **false**?
no
yes
The argument is valid
The argument is invalid (there is a counterexample)

$$\begin{array}{l} A \rightarrow C \\ B \rightarrow C \\ A \vee B \\ \hline \therefore C \end{array}$$

A	B	C	$A \rightarrow C$	$B \rightarrow C$	$A \vee B$
T	T	T			
T	T	F			
T	F	T			
T	F	F			
F	T	T			
F	T	F			
F	F	T			
F	F	F			

$$\begin{array}{l} A \rightarrow C \\ B \rightarrow C \\ A \vee B \\ \hline \therefore C \end{array}$$

A	B	C	$A \rightarrow C$	$B \rightarrow C$	$A \vee B$
T	T	T	T	T	T
T	T	F	F	F	T
T	F	T	T	T	T
T	F	F	F	T	T
F	T	T	T	T	T
F	T	F	T	F	T
F	F	T	T	T	F
F	F	F	T	T	F

$A \to C$
$B \to C$
$A \vee B$

$\therefore C$

A	B	C	$A \to C$	$B \to C$	$A \vee B$
T	T	T	T	T	T
T	T	F	F	F	T
T	F	T	T	T	T
T	F	F	F	T	T
F	T	T	T	T	T
F	T	F	T	F	T
F	F	T	T	T	F
F	F	F	T	T	F

$$\begin{array}{l} A \to C \\ B \to C \\ A \lor B \\ \hline \therefore C \end{array}$$

A	B	C	$A \to C$	$B \to C$	$A \lor B$
T	T	T	T	T	T
T	T	F	F	F	T
T	F	T	T	T	T
T	F	F	F	T	T
F	T	T	T	T	T
F	T	F	T	F	T
F	F	T	T	T	F
F	F	F	T	T	F

$$
\begin{array}{l}
A \rightarrow C \\
B \rightarrow C \\
A \vee B \\
\hline
\therefore C
\end{array}
$$

This argument is valid because there is no truth-value assignment that makes the premises true ($A \rightarrow C$, $B \rightarrow C$ and $A \vee B$) and the conclusion (C) false.

$A \to C$

$B \to C$

$A \lor B$

$\therefore C$

A	B	C	$A \to C$	$B \to C$	$A \lor B$
T	T	T	T	T	T
T	T	F	F	F	T
T	F	T	T	T	T
T	F	F	F	T	T
F	T	T	T	T	T
F	T	F	T	F	T
F	F	T	T	T	F
F	F	F	T	T	F

$A \to C$

$B \to C$

$A \vee B$

$\therefore C$

A	B	C	$A \to C$	$B \to C$	$A \vee B$
T	T	T	T	T	T
T	T	F	F	F	T
T	F	T	T	T	T
T	F	F	F	T	T
F	T	T	T	T	T
F	T	F	T	F	T
F	F	T	T	T	F
F	F	F	T	T	F

$$
\begin{array}{l}
A \to C \\
B \to C \\
A \lor B \\
\hline
\therefore C
\end{array}
$$

This argument is valid because in every row in which the conclusion (C) is false, at least one of the premises ($A \to C$, $B \to C$ or $A \lor B$) is false.

Is the following argument valid or invalid? You must show your answer.

$$\begin{array}{l} (A \vee B) \\ (B \rightarrow C) \\ \hline \therefore (B \wedge C) \end{array}$$

$$(A \vee B)$$
$$(B \to C)$$
$$\therefore (B \wedge C)$$

A	B	C	$(A \vee B)$	$(B \to C)$	$(B \wedge C)$
T	T	T	T	T	T
T	T	F	T	F	F
T	F	T	T	T	F
T	F	F	T	T	F
F	T	T	T	T	T
F	T	F	T	F	F
F	F	T	F	T	F
F	F	F	F	T	F

$$\frac{\begin{array}{l}(A \vee B)\\(B \to C)\end{array}}{\therefore (B \wedge C)}$$

A	B	C	$(A \vee B)$	$(B \to C)$	$(B \wedge C)$
T	T	T	T	T	T
T	T	F	T	F	F
T	F	T	T	T	F
T	F	F	T	T	F
F	T	T	T	T	T
F	T	F	T	F	F
F	F	T	F	T	F
F	F	F	F	T	F

The argument is invalid, because there is a counterexample.

Determine if the following arguments are valid or invalid. (You must explain your answers.)

1. $(A \vee B), (B \to C) \Rightarrow (B \wedge C)$
2. $(A \to (B \to C)), (B \wedge C) \Rightarrow \neg\neg A$
3. $((A \wedge B) \vee (A \to \neg B)), (B \to C) \Rightarrow (\neg C \to A)$
4. $((A \wedge B) \to (B \wedge C)), (B \wedge D) \Rightarrow (A \to (C \to \neg D))$