Reasoning for Humans: Clear Thinking in an Uncertain World

PHIL 171

Eric Pacuit

Department of Philosophy
University of Maryland
pacuit.org

Recap: Truth Tables

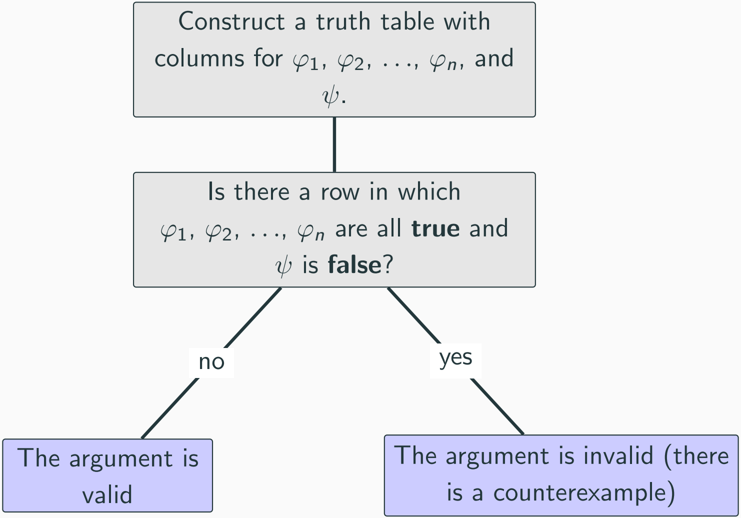
φ	ψ	$(\varphi \wedge \psi)$
T	T	T
T	F	F
F	T	F
F	F	F

φ	ψ	$(\varphi \vee \psi)$
T	T	T
T	F	T
F	T	T
F	F	F

φ	ψ	$(\varphi \rightarrow \psi)$
T	T	T
T	F	F
F	T	T
F	F	T

φ	ψ	$(\varphi \leftrightarrow \psi)$
T	T	T
T	F	F
F	T	F
F	F	T

φ	$\neg\varphi$
T	F
F	T


Construct a truth table with columns for φ_1, φ_2, ..., φ_n, and ψ.
Is there a row in which φ_1, φ_2, ..., φ_n are all **true** and ψ is **false**?
no
yes
The argument is valid
The argument is invalid (there is a counterexample)

Weird Cases: Which arguments are valid?

$$B \Rightarrow (A \vee \neg A)$$

$$A \vee \neg A \Rightarrow B$$

$$A \wedge \neg A \Rightarrow B$$

Weird Cases

- Suppose that an argument contains a tautology as a premise. Is the argument valid?

Weird Cases

- Suppose that an argument contains a tautology as a premise. Is the argument valid? Maybe.

Weird Cases

- Suppose that an argument contains a tautology as a premise. Is the argument valid? Maybe.
- Suppose that the argument contains a contradiction as a premise. Is the argument valid?

Weird Cases

- Suppose that an argument contains a tautology as a premise. Is the argument valid? Maybe.

- Suppose that the argument contains a contradiction as a premise. Is the argument valid? Yes.

Weird Cases

- Suppose that an argument contains a tautology as a premise. Is the argument valid? Maybe.
- Suppose that the argument contains a contradiction as a premise. Is the argument valid? Yes.
- Suppose that the argument contains a tautology as a conclusion. Is the argument valid?

Weird Cases

- Suppose that an argument contains a tautology as a premise. Is the argument valid? Maybe.
- Suppose that the argument contains a contradiction as a premise. Is the argument valid? Yes.
- Suppose that the argument contains a tautology as a conclusion. Is the argument valid? Yes.

Weird Cases

- Suppose that an argument contains a tautology as a premise. Is the argument valid? Maybe.
- Suppose that the argument contains a contradiction as a premise. Is the argument valid? Yes.
- Suppose that the argument contains a tautology as a conclusion. Is the argument valid? Yes.
- Suppose that the argument contains a contradiction as a conclusion. Is the argument valid?

Weird Cases

- Suppose that an argument contains a tautology as a premise. Is the argument valid? Maybe.
- Suppose that the argument contains a contradiction as a premise. Is the argument valid? Yes.
- Suppose that the argument contains a tautology as a conclusion. Is the argument valid? Yes.
- Suppose that the argument contains a contradiction as a conclusion. Is the argument valid? Maybe.

$\varphi_1, \varphi_2, \ldots, \varphi_n \Rightarrow \psi$ denotes an argument with premises $\varphi_1, \ldots, \varphi_n$ and conclusion ψ.

$\varphi_1, \varphi_2, \ldots, \varphi_n \models \psi$ means that the argument is valid.

$\varphi_1, \varphi_2, \ldots, \varphi_n \not\models \psi$ means that the argument is invalid.

$$P, P \to Q \Rightarrow Q$$

$$A, A \to B \Rightarrow B$$

$$Q, Q \to P \Rightarrow P$$

$$\neg A, \neg A \to B \Rightarrow B$$

$$(Q \vee R), (Q \vee R) \to P \Rightarrow P$$

$$(P \to Q), (P \to Q) \to (R \vee (Q \to S)) \Rightarrow (R \vee (Q \to S))$$

$P, P \to Q \models Q$

$A, A \to B \models B$

$Q, Q \to P \models P$

$\neg A, \neg A \to B \models B$

$(Q \vee R), (Q \vee R) \to P \models P$

$(P \to Q), (P \to Q) \to (R \vee (Q \to S)) \models (R \vee (Q \to S))$

$$\varphi, \varphi \to \psi \models \psi$$

Every way of replacing φ with a formula and ψ with a formula results in a valid argument.

Name	Valid inference rule
Modus Ponens	$\varphi, \varphi \to \psi \models \psi$
Modus Tollens	$\varphi \to \psi, \neg\psi \models \neg\varphi$
Disjunctive Syllogism	$\varphi \lor \psi, \neg\varphi \models \psi$
Transitivity	$\varphi \to \psi, \psi \to \chi \models \varphi \to \chi$

$Q, P \to Q \Rightarrow P$

$B, A \to B \Rightarrow A$

$P, Q \to P \Rightarrow Q$

$B, \neg A \to B \Rightarrow \neg A$

$P, (Q \vee R) \to P \Rightarrow (Q \vee R)$

$(R \vee (Q \to S)), (P \to Q) \to (R \vee (Q \to S)) \Rightarrow (P \to Q)$

$Q, P \to Q \not\models P$

$B, A \to B \not\models A$

$P, Q \to P \not\models Q$

$B, \neg A \to B \not\models \neg A$

$P, (Q \vee R) \to P \not\models (Q \vee R)$

$(R \vee (Q \to S)), (P \to Q) \to (R \vee (Q \to S)) \not\models (P \to Q)$

$$\psi, \varphi \to \psi \not\models \varphi$$

Some way of replacing φ with a formula and ψ with a formula results in an invalid argument.

Name	Invalid inference rule
Denying the Antecedent	$\neg\varphi, \varphi \to \psi \not\models \neg\psi$
Affirming the Consequent	$\psi, \varphi \to \psi \not\models \varphi$
Affirming a Disjunct	$\varphi \vee \psi, \varphi \not\models \neg\psi$

$$(Q \wedge P), (P \wedge Q) \to S \Rightarrow S$$

$$(Q \wedge P) \approx (P \wedge Q)$$

$$Q, P \to \neg Q \Rightarrow \neg P$$

$$Q \approx \neg\neg Q$$

$$P, \neg P \vee Q \Rightarrow Q$$

$$\neg P \vee Q \approx P \to Q$$

$$(Q \wedge P), (P \wedge Q) \rightarrow S \models S$$

$$(Q \wedge P) \approx (P \wedge Q)$$

$$Q, P \rightarrow \neg Q \models \neg P$$

$$Q \approx \neg\neg Q$$

$$P, \neg P \vee Q \models Q$$

$$\neg P \vee Q \approx P \rightarrow Q$$

Observation. If φ and ψ are tautologically equivalent, denoted $\varphi \approx \psi$, then for all formulas χ,

$\varphi \models \chi$ if and only if $\psi \models \chi$.

$\chi \models \varphi$ if and only if $\chi \models \psi$.

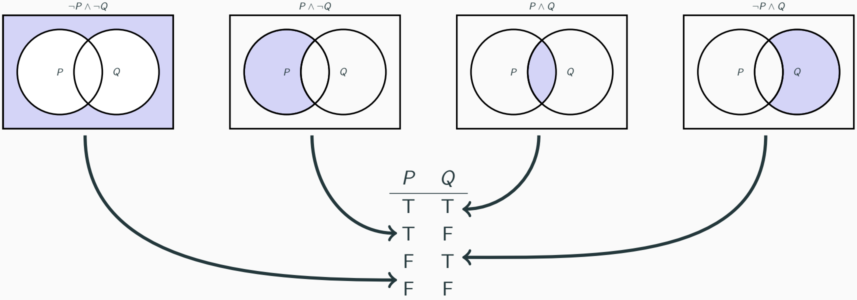
P	Q	$\neg\neg Q$
T	T	T
T	F	F
F	T	T
F	F	F

P	Q	$P \wedge Q$	$Q \wedge P$
T	T	T	T
T	F	F	F
F	T	F	F
F	F	F	F

P	Q	$\neg P \vee Q$	$P \rightarrow Q$
T	T	T	T
T	F	F	F
F	T	T	T
F	F	T	T

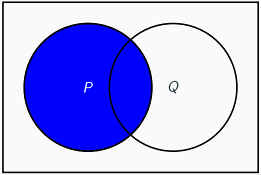
P	Q	$P \wedge Q$	$P \wedge \neg Q$
T	T	T	F
T	F	F	T
F	T	F	F
F	F	F	F

$$P \approx (P \wedge Q) \vee (P \wedge \neg Q)$$

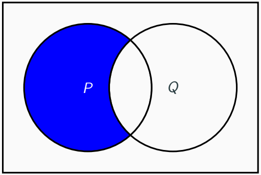


P	Q
T	T
T	F
F	T
F	F

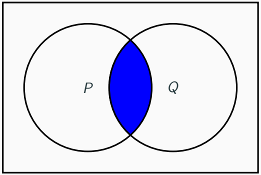
P



$P \wedge \neg Q$



$P \wedge Q$



P	Q	R
T	T	T
T	T	F
T	F	T
T	F	F
F	T	T
F	T	F
F	F	T
F	F	F