Reasoning for Humans: Clear Thinking in an Uncertain World

PHIL 171

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Recap: Truth Tables

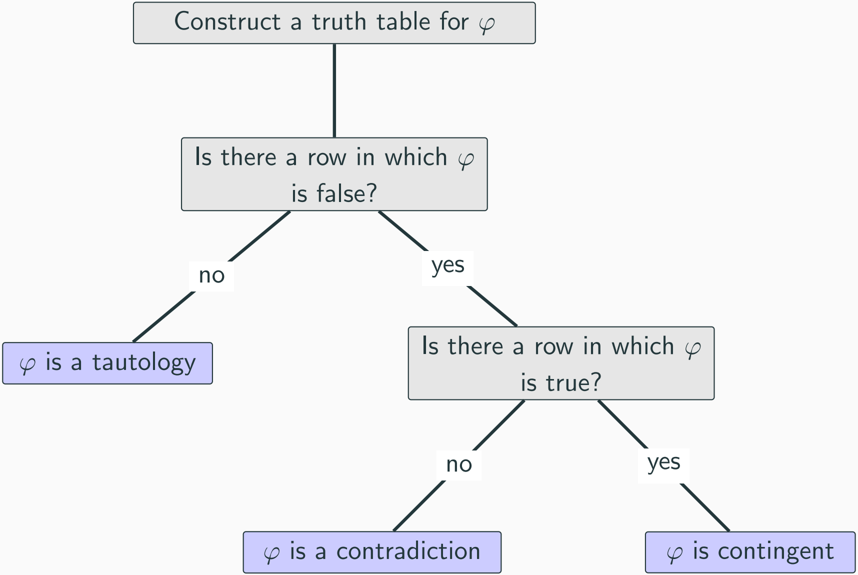
φ	ψ	$(\varphi \wedge \psi)$
T	T	T
T	F	F
F	T	F
F	F	F

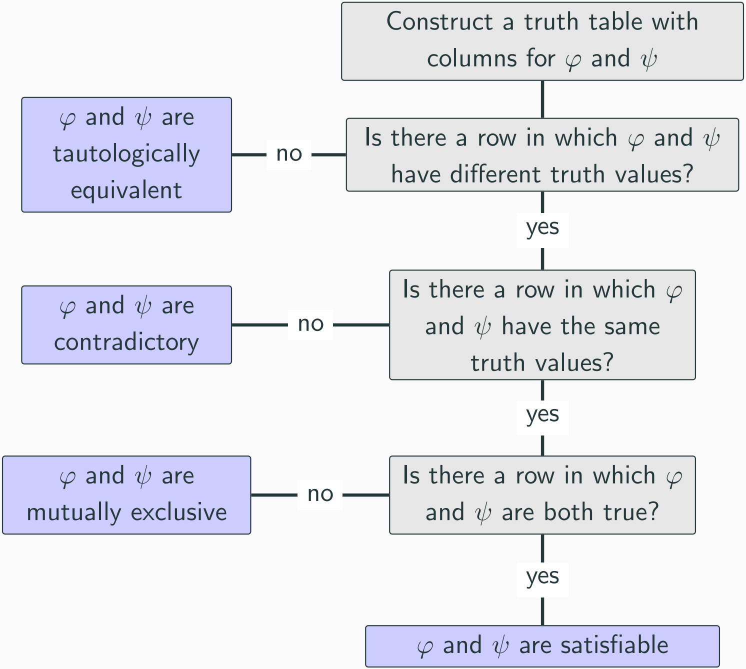
φ	ψ	$(\varphi \vee \psi)$
T	T	T
T	F	T
F	T	T
F	F	F

φ	ψ	$(\varphi \rightarrow \psi)$
T	T	T
T	F	F
F	T	T
F	F	T

φ	ψ	$(\varphi \leftrightarrow \psi)$
T	T	T
T	F	F
F	T	F
F	F	T

φ	$\neg\varphi$
T	F
F	T


Construct a truth table for φ
Is there a row in which φ is false?
no
yes
φ is a tautology
Is there a row in which φ is true?
no
yes
φ is a contradiction
φ is contingent


Construct a truth table with columns for φ and ψ
Is there a row in which φ and ψ have different truth values?
no
φ and ψ are tautologically equivalent
yes
Is there a row in which φ and ψ have the same truth values?
no
φ and ψ are contradictory
yes
Is there a row in which φ and ψ are both true?
no
φ and ψ are mutually exclusive
yes
φ and ψ are satisfiable

Classifying Arguments

Is it possible that the formulas $(A \wedge B)$ and $(A \vee C)$ can both be true at the same time?

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A	B	C	$(A \wedge B)$	$(A \vee C)$
T	T	T		
T	T	F		
T	F	T		
T	F	F		
F	T	T		
F	T	F		
F	F	T		
F	F	F		

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T	T	T	T	
T	T	F	T	
T	F	T	F	
T	F	F	F	
F	T	T	F	
F	T	F	F	
F	F	T	F	
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T	T	T	T	T
T	T	F	T	T
T	F	T	F	T
T	F	F	F	T
F	T	T	F	T
F	T	F	F	F
F	F	T	F	T
F	F	F	F	F

Is it possible that the formulas $(A \wedge B)$ and $(A \vee C)$ can both be true at the same time? Yes...There are two truth assignments that make both formulas true.

A	B	C	$(A \wedge B)$	$(A \vee C)$
T	T	T	T	T
T	T	F	T	T
T	F	T	F	T
T	F	F	F	T
F	T	T	F	T
F	T	F	F	F
F	F	T	F	T
F	F	F	F	F

Is it possible that the formulas $(\neg A \land B)$ and $(A \lor C)$ and $\neg C$ can all be true at the same time?

Is it possible that the formulas $(\neg A \wedge B)$ and $(A \vee C)$ and $\neg C$ can all be true at the same time?

A	B	C	$\neg A$	$(\neg A \wedge B)$	$(A \vee C)$	$\neg C$
T	T	T				
T	T	F				
T	F	T				
T	F	F				
F	T	T				
F	T	F				
F	F	T				
F	F	F				

Is it possible that the formulas $(\neg A \wedge B)$ and $(A \vee C)$ and $\neg C$ can all be true at the same time?

A	B	C	$\neg A$	$(\neg A \wedge B)$	$(A \vee C)$	$\neg C$
T	T	T	F			
T	T	F	F			
T	F	T	F			
T	F	F	F			
F	T	T	T			
F	T	F	T			
F	F	T	T			
F	F	F	T			

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A	B	C	$\neg A$	$(\neg A \wedge B)$	$(A \vee C)$	$\neg C$
T	T	T	F	F		
T	T	F	F	F		
T	F	T	F	F		
T	F	F	F	F		
F	T	T	T	T		
F	T	F	T	T		
F	F	T	T	F		
F	F	F	T	F		

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A	B	C	$\neg A$	$(\neg A \wedge B)$	$(A \vee C)$	$\neg C$
T	T	T	F	F	T	
T	T	F	F	F	T	
T	F	T	F	F	T	
T	F	F	F	F	T	
F	T	T	T	T	T	
F	T	F	T	T	F	
F	F	T	T	F	T	
F	F	F	T	F	F	

Is it possible that the formulas $(\neg A \wedge B)$ and $(A \vee C)$ and $\neg C$ can all be true at the same time?

A	B	C	$\neg A$	$(\neg A \wedge B)$	$(A \vee C)$	$\neg C$
T	T	T	F	F	T	F
T	T	F	F	F	T	T
T	F	T	F	F	T	F
T	F	F	F	F	T	T
F	T	T	T	T	T	F
F	T	F	T	T	F	T
F	F	T	T	F	T	F
F	F	F	T	F	F	T

Is it possible that the formulas $(\neg A \wedge B)$ and $(A \vee C)$ and $\neg C$ can all be true at the same time? No...there is no row in which all these formulas are true.

A	B	C	$\neg A$	$(\neg A \wedge B)$	$(A \vee C)$	$\neg C$
T	T	T	F	F	T	F
T	T	F	F	F	T	T
T	F	T	F	F	T	F
T	F	F	F	F	T	T
F	T	T	T	T	T	F
F	T	F	T	F	F	T
F	F	T	T	F	T	F
F	F	F	T	F	F	T

Valid Argument:

Valid Argument: An argument is valid provided that there is no truth value assignment that makes all the premises true and the conclusion false.

Valid Argument: An argument is valid provided that there is no truth value assignment that makes all the premises true and the conclusion false. (So, any truth-value assignment that makes all the premises true also makes the conclusion true).

Invalid Argument:

Valid Argument: An argument is valid provided that there is no truth value assignment that makes all the premises true and the conclusion false. (So, any truth-value assignment that makes all the premises true also makes the conclusion true).

Invalid Argument: An argument is invalid just in case it is not valid, i.e., if there is some truth-value assignment that makes the premises true and the conclusion false.

Counterexample: A truth-value assignment that makes the premises of an argument true and its conclusion false is called a counterexample to the argument.

Valid Argument: An argument is valid provided that there is no truth value assignment that makes all the premises true and the conclusion false. (So, any truth-value assignment that makes all the premises true also makes the conclusion true).

Invalid Argument: An argument is invalid just in case it is not valid, i.e., if there is some truth-value assignment that makes the premises true and the conclusion false.

Counterexample: A truth-value assignment that makes the premises of an argument true and its conclusion false is called a counterexample to the argument.

So, an argument if valid if there are no counterexamples.

$$\begin{array}{l} A \rightarrow B \\ A \\ \hline \therefore B \end{array}$$

Is this argument valid?

$$\begin{array}{l} A \to B \\ A \\ \hline \therefore B \end{array}$$

Is this argument valid? Yes.

$$\begin{array}{l} A \to B \\ A \\ \hline \therefore B \end{array}$$

Is this argument valid? Yes. Why?

Modus Ponens

$$\begin{array}{l} A \rightarrow B \\ A \\ \hline \therefore\ B \end{array}$$

A	B	$A \rightarrow B$
T	T	T
T	F	F
F	T	T
F	F	T

Modus Ponens

$$\begin{array}{l} A \to B \\ A \\ \hline \therefore\ B \end{array}$$

A	B	$A \to B$
T	T	T
T	F	F
F	T	T
F	F	T

Modus Ponens

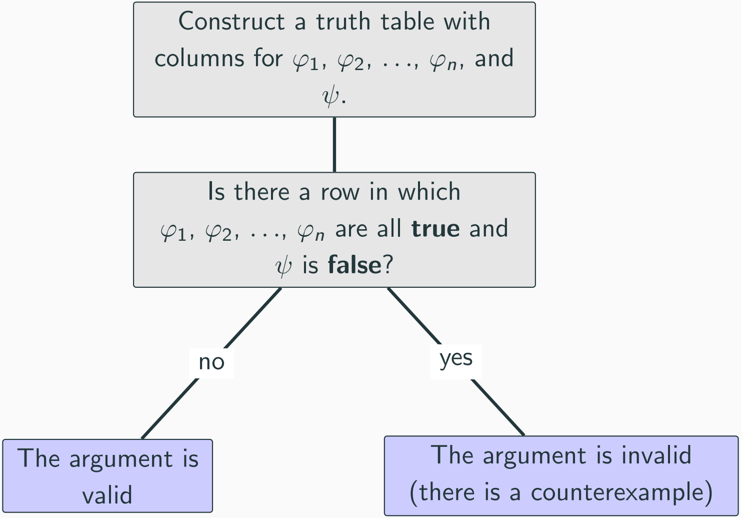
$$A \to B$$
$$A$$
$$\therefore\ B$$

A	B	$A \to B$
T	T	T
T	F	F
F	T	T
F	F	T

Modus Ponens

$$\begin{array}{l} A \to B \\ A \\ \hline \therefore\ B \end{array}$$

Modus Ponens is valid because there is no truth-value assignment that makes the premises true (A, $A \to B$) and the conclusion (B) false.


Construct a truth table with columns for φ_1, φ_2, ..., φ_n, and ψ.
Is there a row in which φ_1, φ_2, ..., φ_n are all **true** and ψ is **false**?
no
yes
The argument is valid
The argument is invalid (there is a counterexample)

$$\begin{array}{l} A \to B \\ B \\ \hline \therefore A \end{array}$$

Is this argument valid?

$$\begin{array}{l} A \rightarrow B \\ B \\ \hline \therefore A \end{array}$$

Is this argument valid? No.

$$\begin{array}{l} A \to B \\ B \\ \hline \therefore A \end{array}$$

Is this argument valid? No.Why?

Affirming the Consequent

$$\begin{array}{l} A \to B \\ B \\ \hline \therefore A \end{array}$$

A	B	$A \to B$
T	T	T
T	F	F
F	T	T
F	F	T

Affirming the Consequent

$$\begin{array}{l} A \rightarrow B \\ B \\ \hline \therefore A \end{array}$$

A	B	$A \rightarrow B$
T	T	T
T	F	F
F	T	T
F	F	T

Affirming the Consequent

$$A \rightarrow B$$
$$B$$
$$\therefore A$$

A	B	$A \rightarrow B$
T	T	T
T	F	F
F	T	T
F	F	T

Affirming the Consequent

$$\begin{array}{l} A \to B \\ B \\ \hline \therefore A \end{array}$$

Affirming the Consequent is not valid because there is a truth-value assigment that makes the premises true and the conclusion false. Namely, the truth-value function that sets A to F and B to T.

Disjunctive Syllogism

$$\begin{array}{l} A \vee B \\ \neg A \\ \hline \therefore B \end{array}$$

A	B	$\neg A$	$A \vee B$
T	T	F	T
T	F	F	T
F	T	T	T
F	F	T	F

Disjunctive Syllogism

$$\begin{array}{l} A \vee B \\ \neg A \\ \hline \therefore B \end{array}$$

A	B	$\neg A$	$A \vee B$
T	T	F	T
T	F	F	T
F	T	T	T
F	F	T	F

Disjunctive Syllogism

$$A \lor B$$
$$\neg A$$
$$\therefore B$$

A	B	$\neg A$	$A \lor B$
T	T	F	T
T	F	F	T
F	T	T	T
F	F	T	F

Disjunctive Syllogism

$$\begin{array}{l} A \vee B \\ \neg A \\ \hline \therefore B \end{array}$$

Disjunctive Syllogism is valid because there is no truth-value assignment that make the premises true ($\neg A$ and $A \vee B$) and the conclusion (B) false.