Reasoning for Humans: Clear Thinking in an Uncertain World

PHIL 171

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Recap: Truth Tables

φ	ψ	$(\varphi \wedge \psi)$
T	T	T
T	F	F
F	T	F
F	F	F

φ	ψ	$(\varphi \vee \psi)$
T	T	T
T	F	T
F	T	T
F	F	F

φ	ψ	$(\varphi \rightarrow \psi)$
T	T	T
T	F	F
F	T	T
F	F	T

φ	$\neg\varphi$
T	F
F	T

Find truth tables for the formulas:

- $P \wedge Q$
- $(P \wedge Q) \rightarrow P$
- $(P \wedge Q) \wedge (\neg P \vee \neg Q)$

	P	Q	$(P \wedge Q)$	$(P \wedge Q) \rightarrow P$	$(P \wedge Q) \wedge (\neg P \vee \neg Q)$
1.	T	T	T	T	F
2.	T	F	F	T	F
3.	F	T	F	T	F
4.	F	F	F	T	F

$P \wedge Q$ is T in some rows and F in some rows (**contingent**)

$(P \wedge Q) \rightarrow P$ is T in every row (**tautology**)

$(P \wedge Q) \wedge (\neg P \vee \neg Q)$ is F in every row (**contradiction**)

	P	Q	$(P \wedge Q)$	$(P \wedge Q) \rightarrow P$	$(P \wedge Q) \wedge (\neg P \vee \neg Q)$
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2.	T	F	F	T	F
3.	F	T	F	T	F
4.	F	F	F	T	F

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$(P \wedge Q) \rightarrow P$ is T in every row (**tautology**)

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Classifying Formulas

A **truth-table** for φ specifies the truth-value of φ for every possible truth-value assignment.

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A formula is a **tautology** provided that it is true under all truth-value assignments (the only truth value in the column under the formula is T).

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A formula is **contradictory** provided that it is false under all truth-value assignments (the only truth value in the column under the formula is F).

A formula is **contingent** provided that

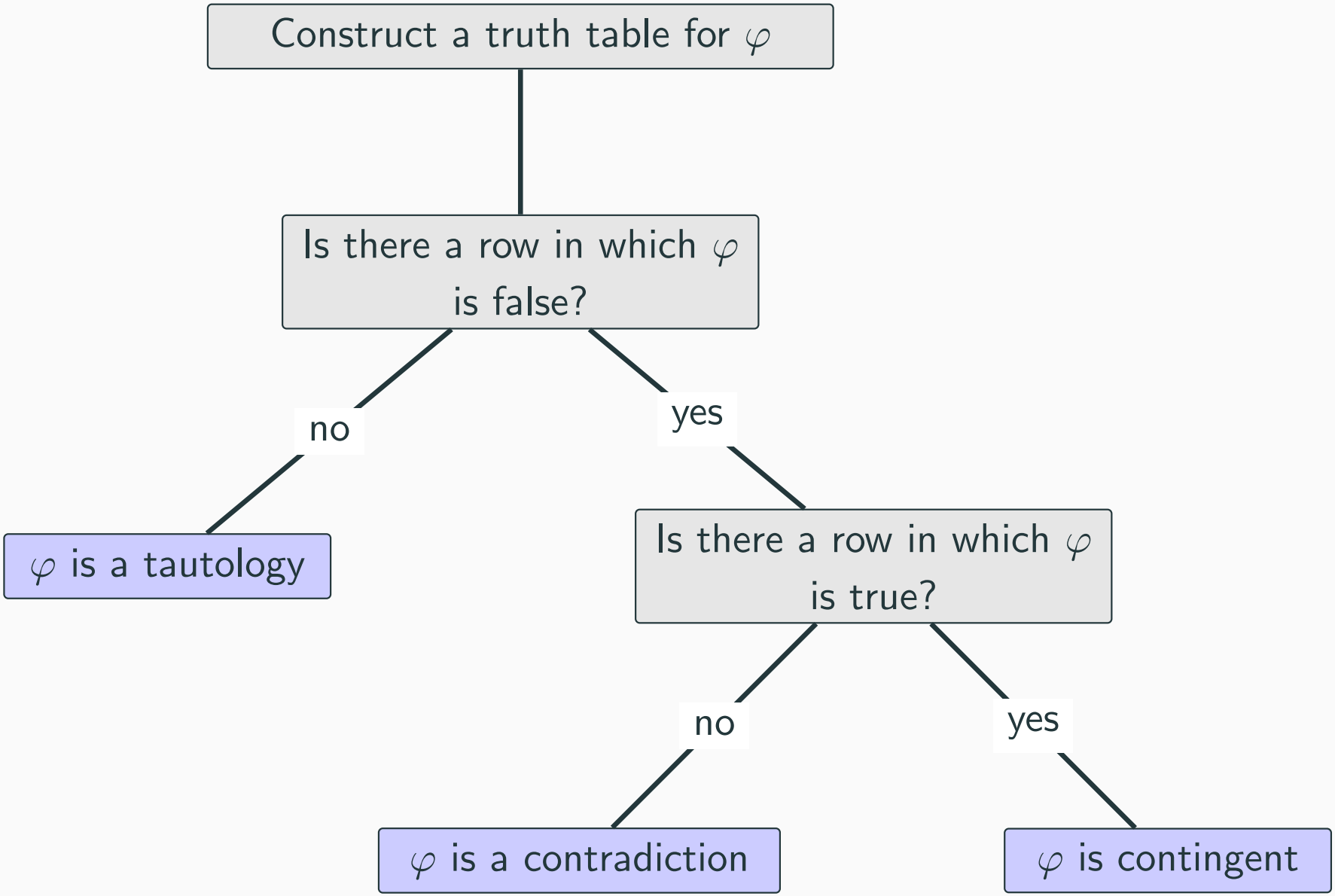
Classifying Formulas

A **truth-table** for φ specifies the truth-value of φ for every possible truth-value assignment.

A formula is a **tautology** provided that it is true under all truth-value assignments (the only truth value in the column under the formula is T).

A formula is **contradictory** provided that it is false under all truth-value assignments (the only truth value in the column under the formula is F).

A formula is **contingent** provided that it is true under some truth-value assignments and false under other truth-value assignments (there are both T and F in the column under the formula).


Construct a truth table for φ
Is there a row in which φ is false?
no
yes
φ is a tautology
Is there a row in which φ is true?
no
yes
φ is a contradiction
φ is contingent

Find truth tables for the formulas

- $P \wedge Q$
- $\neg(P \wedge Q)$
- $\neg P \vee \neg Q$
- $\neg P \wedge \neg Q$

	P	Q	$(P \wedge Q)$	$\neg(P \wedge Q)$	$(\neg P \vee \neg Q)$	$(\neg P \wedge \neg Q)$
1.	T	T	T	F	F	F
2.	T	F	F	T	T	F
3.	F	T	F	T	T	F
4.	F	F	F	T	T	T

$(P \wedge Q)$ and $\neg(P \wedge Q)$ are **contradictory**: they always have opposite truth values

	P	Q	$(P \wedge Q)$	$\neg(P \wedge Q)$	$(\neg P \vee \neg Q)$	$(\neg P \wedge \neg Q)$
1.	T	T	T	F	F	F
2.	T	F	F	T	T	F
3.	F	T	F	T	T	F
4.	F	F	F	T	T	T

$(P \wedge Q)$ and $\neg(P \wedge Q)$ are **contradictory**: they always have opposite truth values

	P	Q	$(P \wedge Q)$	$\neg(P \wedge Q)$	$(\neg P \vee \neg Q)$	$(\neg P \wedge \neg Q)$
1.	T	T	T	F	F	F
2.	T	F	F	T	T	F
3.	F	T	F	T	T	F
4.	F	F	F	T	T	T

$\neg(P \wedge Q)$ are $(\neg P \vee \neg Q)$ are **tautologically equivalent**: they always have the same truth values

	P	Q	$(P \wedge Q)$	$\neg(P \wedge Q)$	$(\neg P \vee \neg Q)$	$(\neg P \wedge \neg Q)$
1.	T	T	T	F	F	F
2.	T	F	F	T	T	F
3.	F	T	F	T	T	F
4.	F	F	F	T	T	T

$(P \wedge Q)$ are $(\neg P \wedge \neg Q)$ are **mutually exclusive**: they are never true in the same situation, but may be false in the same situation

	P	Q	$(P \wedge Q)$	$\neg(P \wedge Q)$	$(\neg P \vee \neg Q)$	$(\neg P \wedge \neg Q)$
1.	T	T	T	F	F	F
2.	T	F	F	T	T	F
3.	F	T	F	T	T	F
4.	F	F	F	T	T	T

$(\neg P \vee \neg Q)$ are $(\neg P \wedge \neg Q)$ are **satisfiable**: there is at least on situation in which both are true.

Classifying Two Formulas

Suppose that φ and ψ are two formulas.

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Suppose that φ and ψ are two formulas.

φ and ψ are **tautologically equivalent** provided that every truth-value assignment assigns the same truth value to φ and ψ.

φ and ψ are **contradictory** provided that every truth-value assignment assigns different truth values to φ and ψ.

φ and ψ are **mutually exclusive** provided that no truth-value assignment assigns true to both φ and ψ (though they may both be false).

φ and ψ are **satisfiable** provided that there is a truth-value assignment that assigns true to both φ and ψ.

Does every truth-value assignment assign the same truth value to φ and ψ?

If yes, then φ and ψ are tautologically equivalent.

Does every truth-value assignment assign different truth values to φ and ψ?

If yes, then φ and ψ are contradictory.

Does every truth-value assignment assign the same truth value to φ and ψ?

If yes, then φ and ψ are tautologically equivalent.

Does every truth-value assignment assign different truth values to φ and ψ?

If yes, then φ and ψ are contradictory.

Does every truth-value assignment assign the same truth value to φ and ψ?

If yes, then φ and ψ are tautologically equivalent.

Does every truth-value assignment assign different truth values to φ and ψ?

If yes, then φ and ψ are contradictory.

Is there some truth-value assignment that assigns T to both φ and ψ?

If yes, then φ and ψ are satisfiable.

If no, then φ and ψ are mutually exclusive.

Does every truth-value assignment assign the same truth value to φ and ψ?

If yes, then φ and ψ are tautologically equivalent.

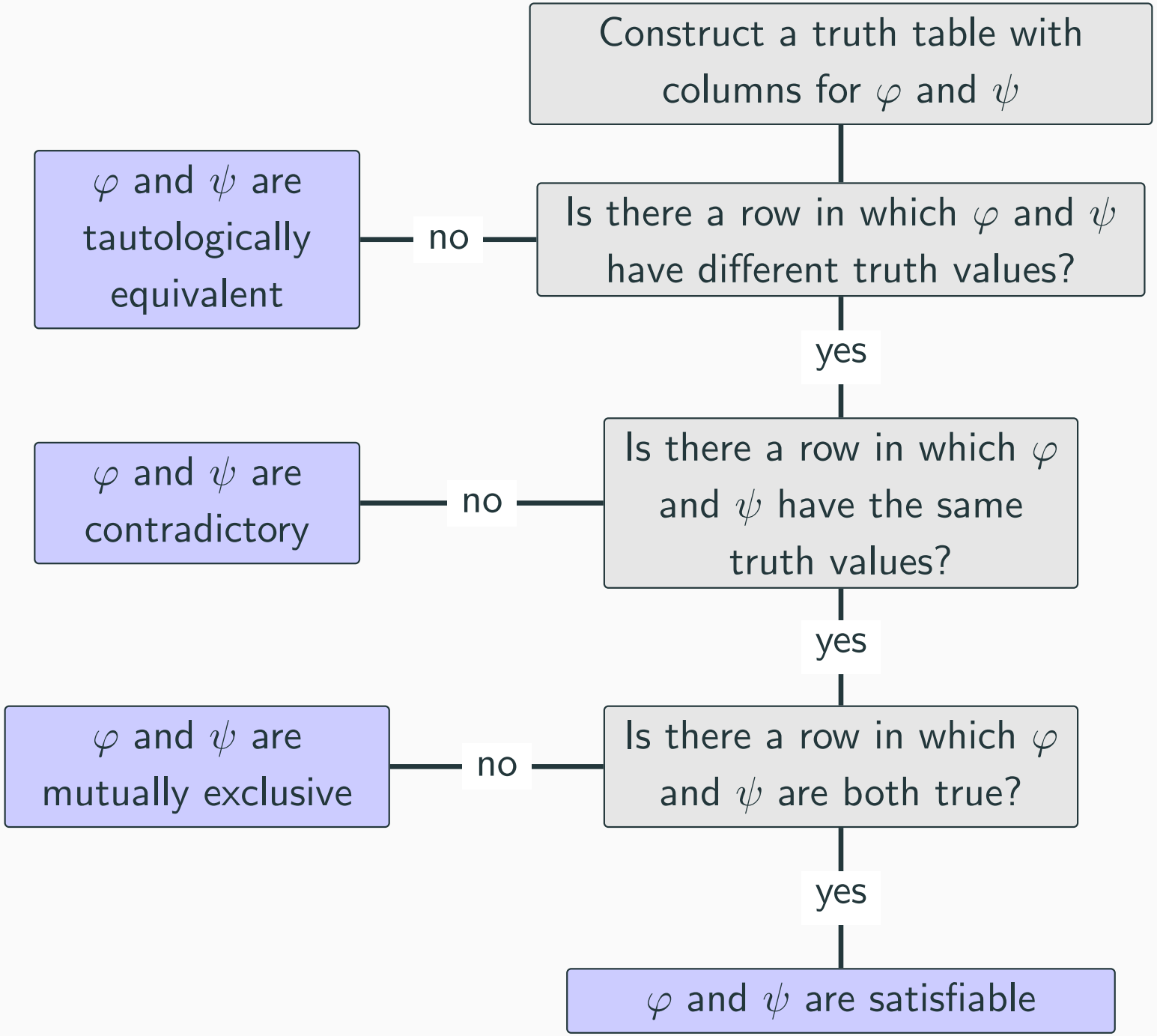
Does every truth-value assignment assign different truth values to φ and ψ?

If yes, then φ and ψ are contradictory.

Is there some truth-value assignment that assigns T to both φ and ψ?

If yes, then φ and ψ are satisfiable.

If no, then φ and ψ are mutually exclusive.


Construct a truth table with columns for φ and ψ
Is there a row in which φ and ψ have different truth values?
no
φ and ψ are tautologically equivalent
yes
Is there a row in which φ and ψ have the same truth values?
no
φ and ψ are contradictory
yes
Is there a row in which φ and ψ are both true?
no
φ and ψ are mutually exclusive
yes
φ and ψ are satisfiable

Bi-conditional

Recall that $\varphi \leftrightarrow \psi$ is short-hand for $(\varphi \to \psi) \wedge (\psi \to \varphi)$. What is the truth table for $\varphi \leftrightarrow \psi$?

φ	ψ	$(\varphi \leftrightarrow \psi)$
T	T	
T	F	
F	T	
F	F	

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Recall that $\varphi \leftrightarrow \psi$ is short-hand for $(\varphi \rightarrow \psi) \wedge (\psi \rightarrow \varphi)$. What is the truth table for $\varphi \leftrightarrow \psi$?

φ	ψ	$(\varphi \leftrightarrow \psi)$
T	T	T
T	F	
F	T	
F	F	T

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T	F	F
F	T	F
F	F	T