Reasoning for Humans: Clear Thinking in an Uncertain World

PHIL 171

Eric Pacuit

Department of Philosophy
University of Maryland
pacuit.org

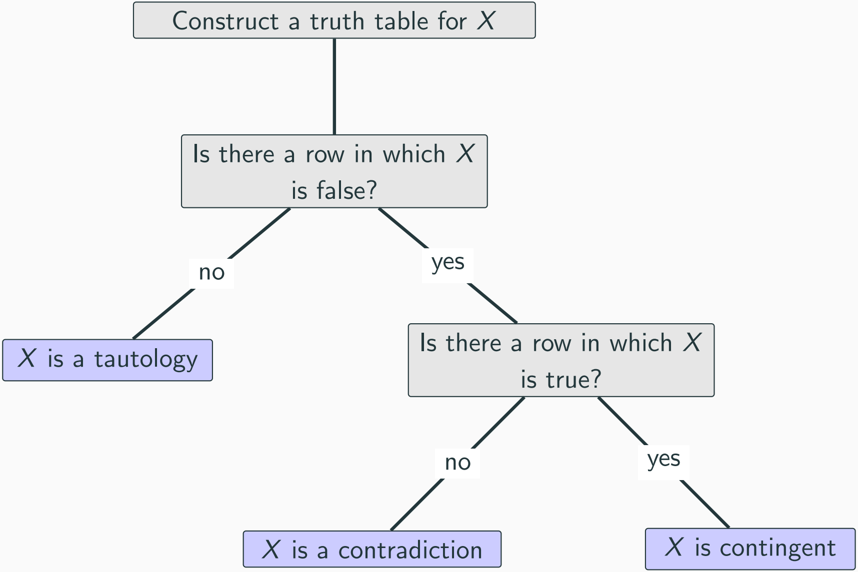
Recap: Truth Tables

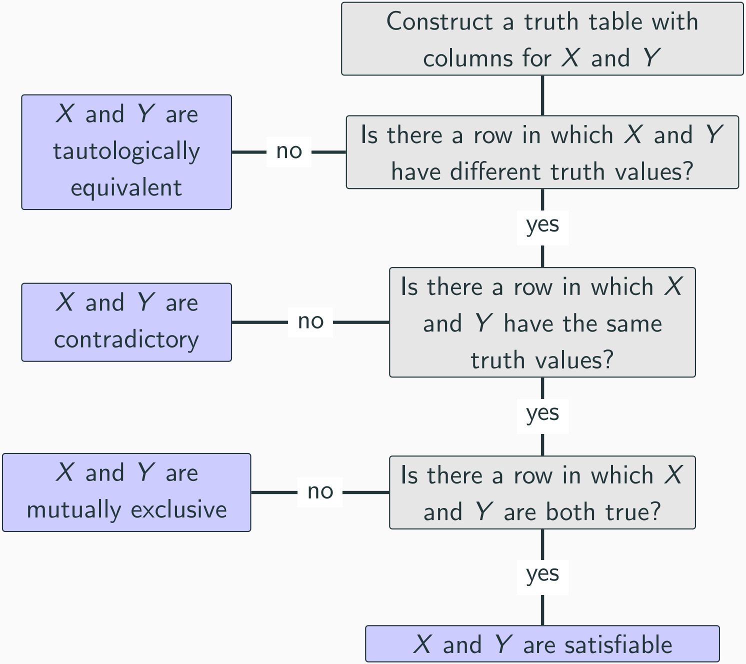
X	Y	$(X \wedge Y)$
T	T	T
T	F	F
F	T	F
F	F	F

X	Y	$(X \vee Y)$
T	T	T
T	F	T
F	T	T
F	F	F

X	Y	$(X \rightarrow Y)$
T	T	T
T	F	F
F	T	T
F	F	T

X	$\neg X$
T	F
F	T


Construct a truth table for X
Is there a row in which X is false?
no
yes
X is a tautology
Is there a row in which X is true?
no
yes
X is a contradiction
X is contingent


Construct a truth table with columns for X and Y
Is there a row in which X and Y have different truth values?
no
X and Y are tautologically equivalent
yes
Is there a row in which X and Y have the same truth values?
no
X and Y are contradictory
yes
Is there a row in which X and Y are both true?
no
X and Y are mutually exclusive
yes
X and Y are satisfiable

Bi-conditional

Suppose that $X \leftrightarrow Y$ is short-hand for $(X \to Y) \wedge (Y \to X)$. What is the truth table for $X \leftrightarrow Y$?

X	Y	$(X \leftrightarrow Y)$
T	T	
T	F	
F	T	
F	F	

Bi-conditional

Suppose that $X \leftrightarrow Y$ is short-hand for $(X \to Y) \wedge (Y \to X)$. What is the truth table for $X \leftrightarrow Y$?

X	Y	$(X \leftrightarrow Y)$
T	T	T
T	F	
F	T	
F	F	T

Bi-conditional

Suppose that $X \leftrightarrow Y$ is short-hand for $(X \rightarrow Y) \wedge (Y \rightarrow X)$. What is the truth table for $X \leftrightarrow Y$?

X	Y	$(X \leftrightarrow Y)$
T	T	T
T	F	F
F	T	F
F	F	T

Classifying Arguments

Is it possible that the formulas $(A \wedge B)$ and $(A \vee C)$ can both be true at the same time?

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A	B	C	$(A \wedge B)$	$(A \vee C)$
T	T	T		
T	T	F		
T	F	T		
T	F	F		
F	T	T		
F	T	F		
F	F	T		
F	F	F		

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A	B	C	$(A \wedge B)$	$(A \vee C)$
T	T	T	T	
T	T	F	T	
T	F	T	F	
T	F	F	F	
F	T	T	F	
F	T	F	F	
F	F	T	F	
F	F	F	F	

Is it possible that the formulas $(A \wedge B)$ and $(A \vee C)$ can both be true at the same time?

A	B	C	$(A \wedge B)$	$(A \vee C)$
T	T	T	T	T
T	T	F	T	T
T	F	T	F	T
T	F	F	F	T
F	T	T	F	T
F	T	F	F	F
F	F	T	F	T
F	F	F	F	F

Is it possible that the formulas $(A \wedge B)$ and $(A \vee C)$ can both be true at the same time? Yes...There are two truth assignments that make both formulas true.

A	B	C	$(A \wedge B)$	$(A \vee C)$
T	T	T	T	T
T	T	F	T	T
T	F	T	F	T
T	F	F	F	T
F	T	T	F	T
F	T	F	F	F
F	F	T	F	T
F	F	F	F	F

Is it possible that the formulas $(\neg A \wedge B)$ and $(A \vee C)$ and $\neg C$ can all be true at the same time?

Is it possible that the formulas $(\neg A \wedge B)$ and $(A \vee C)$ and $\neg C$ can all be true at the same time?

A	B	C	$\neg A$	$(\neg A \wedge B)$	$(A \vee C)$	$\neg C$
T	T	T				
T	T	F				
T	F	T				
T	F	F				
F	T	T				
F	T	F				
F	F	T				
F	F	F				

Is it possible that the formulas $(\neg A \wedge B)$ and $(A \vee C)$ and $\neg C$ can all be true at the same time?

A	B	C	$\neg A$	$(\neg A \wedge B)$	$(A \vee C)$	$\neg C$
T	T	T	F			
T	T	F	F			
T	F	T	F			
T	F	F	F			
F	T	T	T			
F	T	F	T			
F	F	T	T			
F	F	F	T			

Is it possible that the formulas $(\neg A \wedge B)$ and $(A \vee C)$ and $\neg C$ can all be true at the same time?

A	B	C	$\neg A$	$(\neg A \wedge B)$	$(A \vee C)$	$\neg C$
T	T	T	F	F		
T	T	F	F	F		
T	F	T	F	F		
T	F	F	F	F		
F	T	T	T	T		
F	T	F	T	T		
F	F	T	T	F		
F	F	F	T	F		

Is it possible that the formulas $(\neg A \wedge B)$ and $(A \vee C)$ and $\neg C$ can all be true at the same time?

A	B	C	$\neg A$	$(\neg A \wedge B)$	$(A \vee C)$	$\neg C$
T	T	T	F	F	T	
T	T	F	F	F	T	
T	F	T	F	F	T	
T	F	F	F	F	T	
F	T	T	T	T	T	
F	T	F	T	T	F	
F	F	T	T	F	T	
F	F	F	T	F	F	

Is it possible that the formulas $(\neg A \wedge B)$ and $(A \vee C)$ and $\neg C$ can all be true at the same time?

A	B	C	$\neg A$	$(\neg A \wedge B)$	$(A \vee C)$	$\neg C$
T	T	T	F	F	T	F
T	T	F	F	F	T	T
T	F	T	F	F	T	F
T	F	F	F	F	T	T
F	T	T	T	T	T	F
F	T	F	T	T	F	T
F	F	T	T	F	T	F
F	F	F	T	F	F	T

Is it possible that the formulas $(\neg A \wedge B)$ and $(A \vee C)$ and $\neg C$ can all be true at the same time? No...there is no row in which all these formulas are true.

A	B	C	$\neg A$	$(\neg A \wedge B)$	$(A \vee C)$	$\neg C$
T	T	T	F	F	T	F
T	T	F	F	F	T	T
T	F	T	F	F	T	F
T	F	F	F	F	T	T
F	T	T	T	T	T	F
F	T	F	T	F	F	T
F	F	T	T	F	T	F
F	F	F	T	F	F	T

Valid Argument:

Valid Argument: An argument is valid provided that there is no truth value assignment that makes all the premises true and the conclusion false.

Valid Argument: An argument is valid provided that there is no truth value assignment that makes all the premises true and the conclusion false. (So, any truth-value assignment that makes all the premises true also makes the conclusion true).

Invalid Argument:

Valid Argument: An argument is valid provided that there is no truth value assignment that makes all the premises true and the conclusion false. (So, any truth-value assignment that makes all the premises true also makes the conclusion true).

Invalid Argument: An argument is invalid just in case it is not valid, i.e., if there is some truth-value assignment that makes the premises true and the conclusion false.

Counterexample: A truth-value assignment that makes the premises of an argument true and its conclusion false is called a counterexample to the argument.

Valid Argument: An argument is valid provided that there is no truth value assignment that makes all the premises true and the conclusion false. (So, any truth-value assignment that makes all the premises true also makes the conclusion true).

Invalid Argument: An argument is invalid just in case it is not valid, i.e., if there is some truth-value assignment that makes the premises true and the conclusion false.

Counterexample: A truth-value assignment that makes the premises of an argument true and its conclusion false is called a counterexample to the argument.

So, an argument if valid if there are no counterexamples.

Notation

$X_1, \ldots, X_n \Rightarrow Y$:
An argument with premises $X_1, \ldots, X_n$ and conclusion Y.

$X_1, \ldots, X_n \models Y$:
A valid argument with premises $X_1, \ldots, X_n$ and conclusion Y.

$X_1, \ldots, X_n \not\models Y$:
An invalid argument with premises $X_1, \ldots, X_n$ and conclusion Y.

$$\begin{array}{l} A \to B \\ A \\ \hline \therefore B \end{array}$$

Is this argument valid?

$$\begin{array}{l} A \rightarrow B \\ A \\ \hline \therefore B \end{array}$$

Is this argument valid? Yes.

$$\begin{array}{l} A \rightarrow B \\ A \\ \hline \therefore B \end{array}$$

Is this argument valid? Yes. Why?

Modus Ponens

$$\begin{array}{l} A \rightarrow B \\ A \\ \hline \therefore\ B \end{array}$$

A	B	$A \rightarrow B$
T	T	T
T	F	F
F	T	T
F	F	T

Modus Ponens

$$\begin{array}{l} A \to B \\ A \\ \hline \therefore\ B \end{array}$$

A	B	$A \to B$
T	T	T
T	F	F
F	T	T
F	F	T

Modus Ponens

$$\begin{array}{l} A \to B \\ A \\ \hline \therefore\ B \end{array}$$

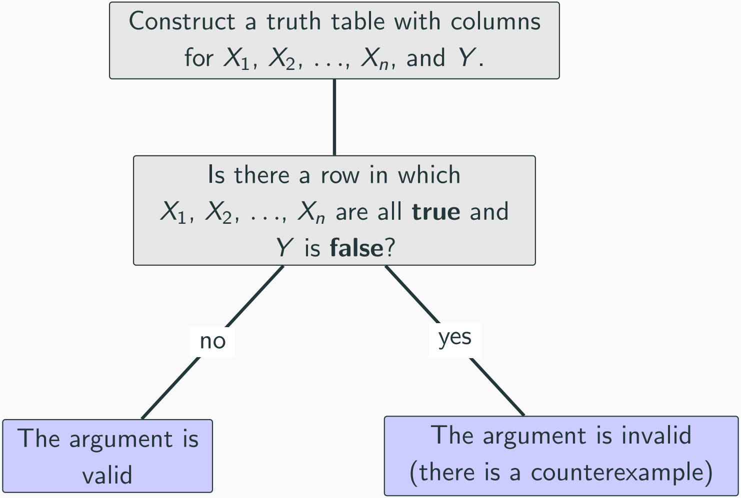
A	B	$A \to B$
T	T	T
T	F	F
F	T	T
F	F	T

Modus Ponens

$$\begin{array}{l} A \rightarrow B \\ A \\ \hline \therefore\ B \end{array}$$

Modus Ponens is valid because there is no truth-value assignment that makes the premises true (A, $A \rightarrow B$) and the conclusion (B) false.

$$A \rightarrow B, A \models B$$


Construct a truth table with columns for X_1, X_2, ..., X_n, and Y.
Is there a row in which X_1, X_2, ..., X_n are all **true** and Y is **false**?
no
yes
The argument is valid
The argument is invalid (there is a counterexample)

$$\begin{array}{l} A \to B \\ B \\ \hline \therefore A \end{array}$$

Is this argument valid?

$$
\begin{array}{l}
A \rightarrow B \\
B \\
\hline
\therefore A
\end{array}
$$

Is this argument valid? No.

$$\begin{array}{l} A \to B \\ B \\ \hline \therefore A \end{array}$$

Is this argument valid? No.Why?

Affirming the Consequent

$$\begin{array}{l} A \rightarrow B \\ B \\ \hline \therefore A \end{array}$$

A	B	$A \rightarrow B$
T	T	T
T	F	F
F	T	T
F	F	T

Affirming the Consequent

$$A \to B$$
$$B$$
$$\therefore A$$

A	B	$A \to B$
T	T	T
T	F	F
F	T	T
F	F	T

Affirming the Consequent

$$A \to B$$
$$B$$
$$\therefore A$$

A	B	$A \to B$
T	T	T
T	F	F
F	T	T
F	F	T

Affirming the Consequent

$$\begin{array}{l} A \to B \\ B \\ \hline \therefore A \end{array}$$

Affirming the Consequent is not valid because there is a truth-value assigment that makes the premises true and the conclusion false. Namely, the truth-value function that sets A to F and B to T.

$$A \to B, B \not\models A$$

Disjunctive Syllogism

$$\begin{array}{l} A \lor B \\ \neg A \\ \hline \therefore B \end{array}$$

A	B	$\neg A$	$A \lor B$
T	T	F	T
T	F	F	T
F	T	T	T
F	F	T	F

Disjunctive Syllogism

$$A \vee B$$
$$\neg A$$
$$\therefore B$$

A	B	$\neg A$	$A \vee B$
T	T	F	T
T	F	F	T
F	T	T	T
F	F	T	F

Disjunctive Syllogism

$$\begin{array}{l} A \vee B \\ \neg A \\ \hline \therefore B \end{array}$$

A	B	$\neg A$	$A \vee B$
T	T	F	T
T	F	F	T
F	T	T	T
F	F	T	F

Disjunctive Syllogism

$$\begin{array}{l} A \vee B \\ \neg A \\ \hline \therefore B \end{array}$$

Disjunctive Syllogism is valid because there is no truth-value assignment that make the premises true ($\neg A$ and $A \vee B$) and the conclusion (B) false.

$$A \vee B, \neg A \models B$$