Reasoning for Humans: Clear Thinking in an Uncertain World

PHIL 171

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Recap: Truth Tables

X	Y	$(X \wedge Y)$
T	T	T
T	F	F
F	T	F
F	F	F

X	Y	$(X \vee Y)$
T	T	T
T	F	T
F	T	T
F	F	F

X	Y	$(X \rightarrow Y)$
T	T	T
T	F	F
F	T	T
F	F	T

X	$\neg X$
T	F
F	T

Suppose that A is T, B is F and C is T.
What is the truth value of $\neg B \to (A \vee \neg C)$?

$$\neg B \to (A \vee \neg C) \text{ is ??}$$

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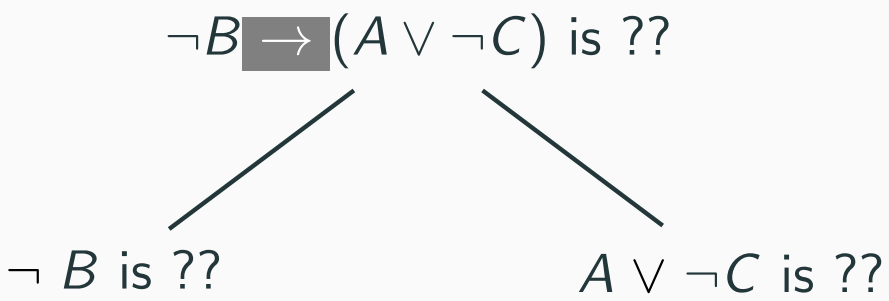

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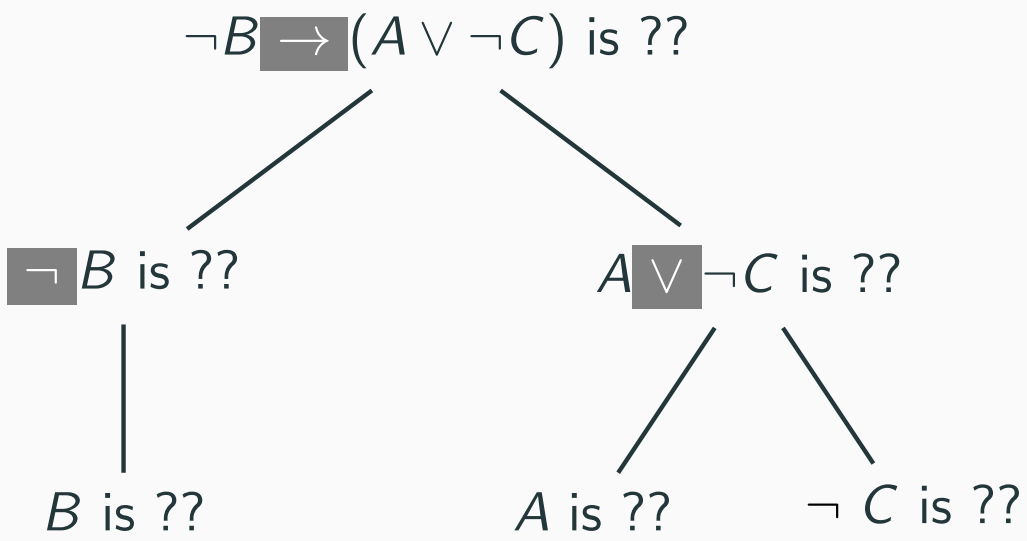

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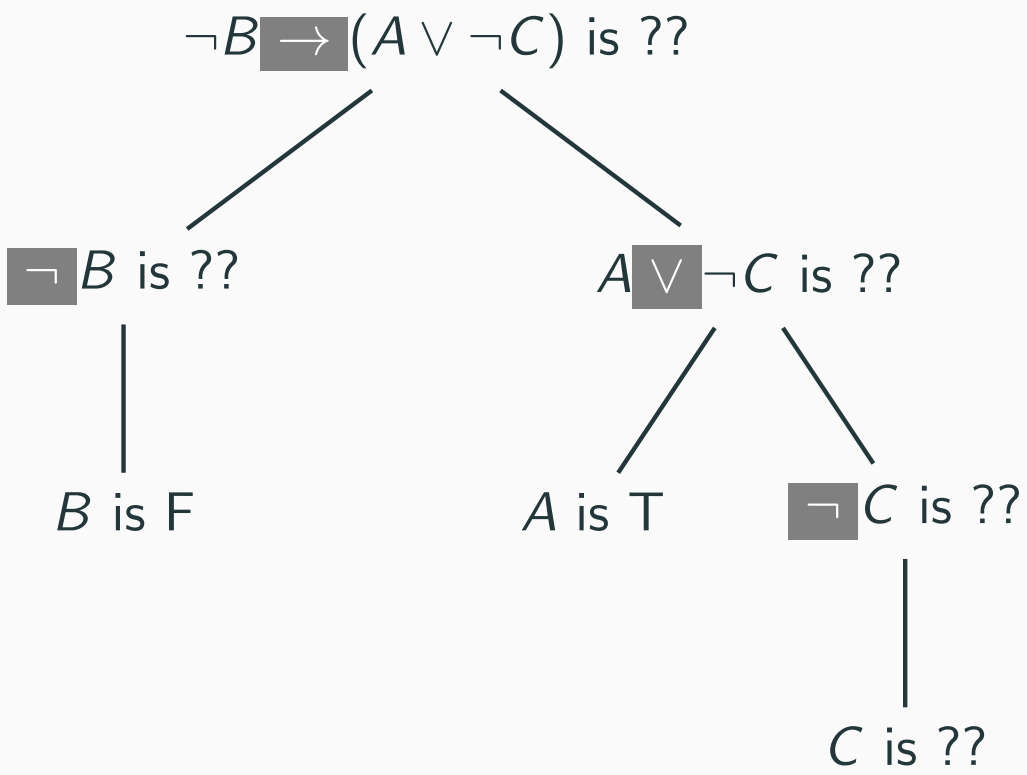

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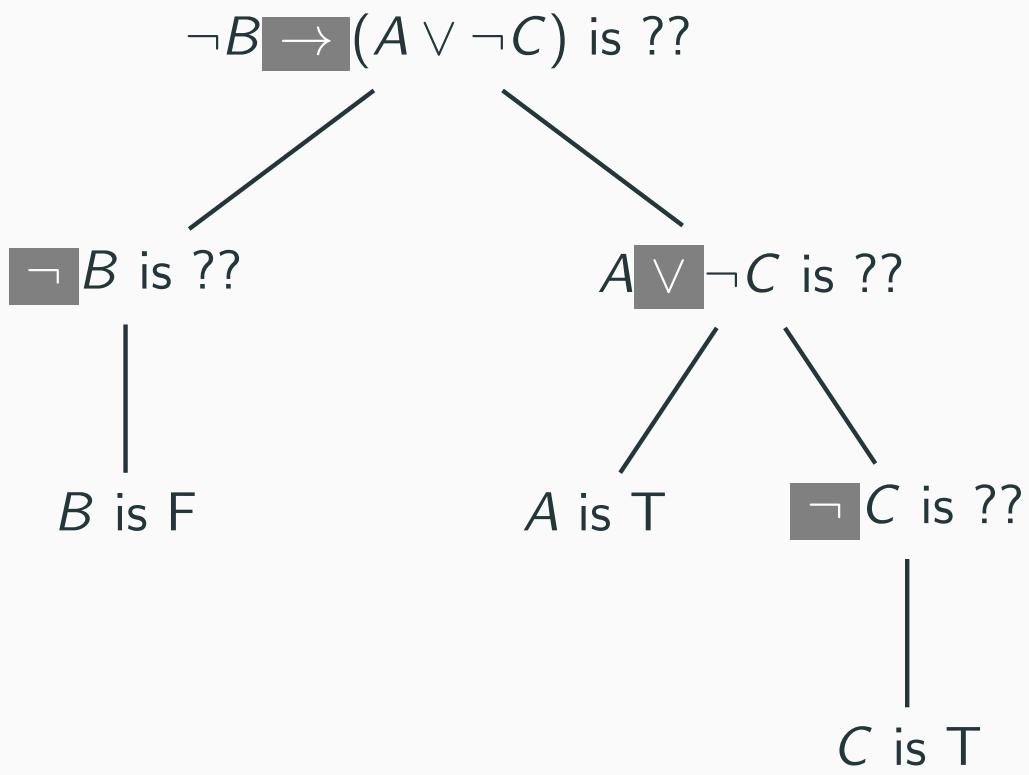

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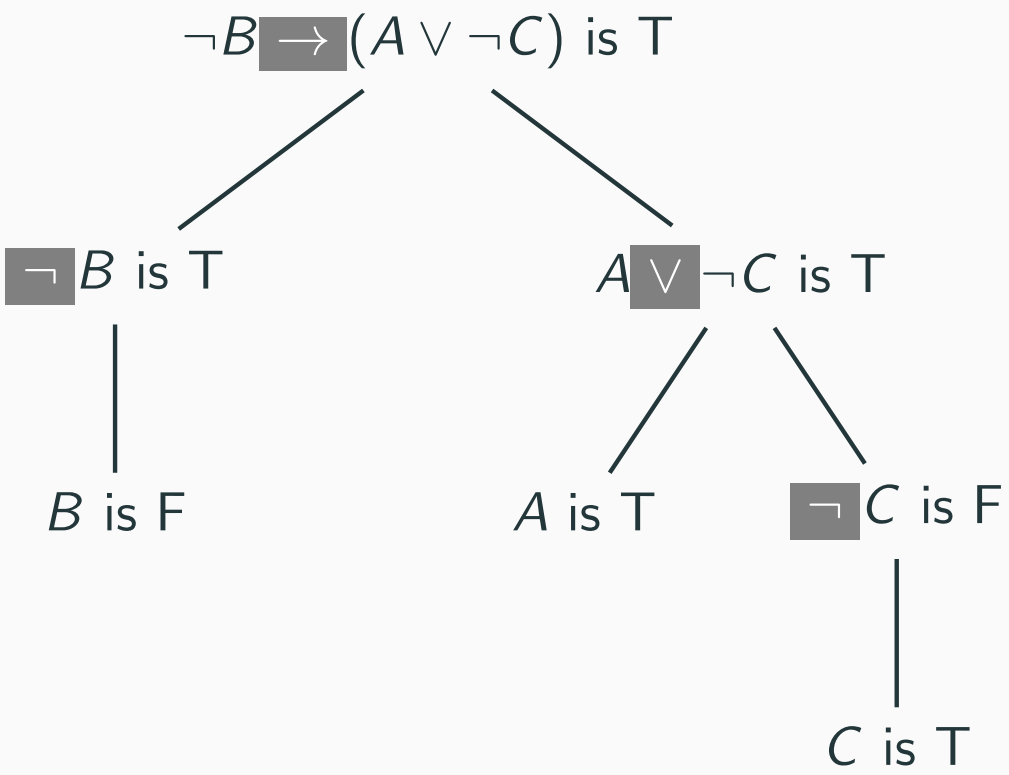

How many truth value assignments are there for a single atomic proposition A?

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How many truth value assignments are there for four atomic propositions A, B, C and D?

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How many truth value assignments are there for n atomic propositions A_1, A_2, $\ldots$, A_n?

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How many truth value assignments are there for n atomic propositions A_1, A_2, ..., A_n? 2^n

A truth table for a formula X is a table, where each row is a truth assignment for the atomic propositions in X and there is a column for X (and possible subformulas of X) list the truth values of X for each truth assignment.

Find truth tables for the formulas:

- $P \wedge Q$
- $(P \wedge Q) \rightarrow P$
- $(P \wedge Q) \wedge (\neg P \vee \neg Q)$

	P	Q	$(P \wedge Q)$	$(P \wedge Q) \rightarrow P$	$(P \wedge Q) \wedge (\neg P \vee \neg Q)$
1.	T	T	T	T	F
2.	T	F	F	T	F
3.	F	T	F	T	F
4.	F	F	F	T	F

$P \wedge Q$ is T in some rows and F in some rows (**contingent**)

$(P \wedge Q) \rightarrow P$ is T in every row (**tautology**)

$(P \wedge Q) \wedge (\neg P \vee \neg Q)$ is F in every row (**contradiction**)

	P	Q	$(P \wedge Q)$	$(P \wedge Q) \rightarrow P$	$(P \wedge Q) \wedge (\neg P \vee \neg Q)$
1.	T	T	T	T	F
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3.	F	T	F	T	F
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Classifying Formulas

A **truth-table** for X specifies the truth-value of X for every possible truth-value assignment.

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A formula is **contingent** provided that

Classifying Formulas

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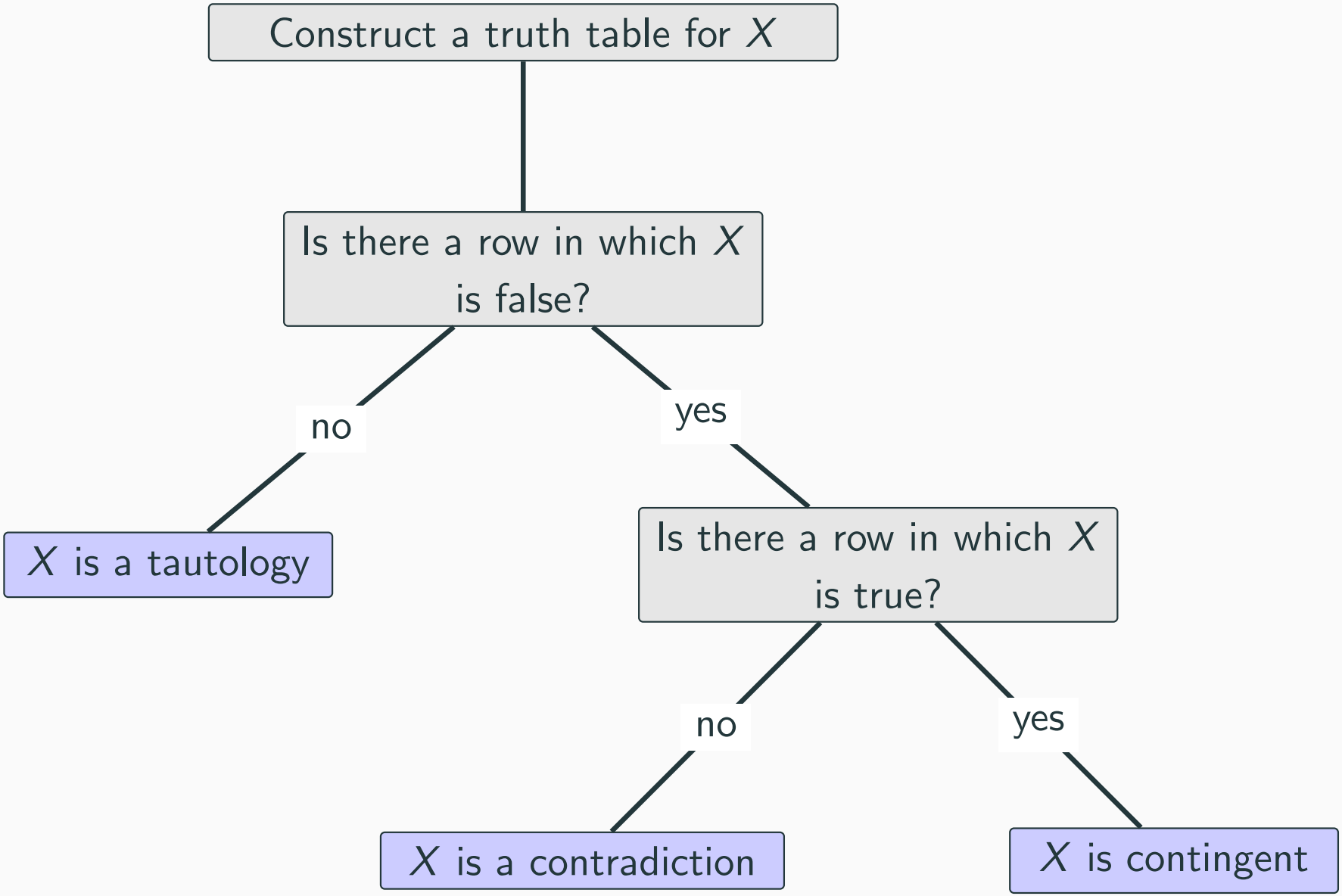
A formula is a **tautology** provided that it is true under all truth-value assignments (the only truth value in the column under the formula is T).

A formula is **contradictory** provided that it is false under all truth-value assignments (the only truth value in the column under the formula is F).

A formula is **contingent** provided that it is true under some truth-value assignments and false under other truth-value assignments (there are both T and F in the column under the formula).


Construct a truth table for X
Is there a row in which X is false?
no
yes
X is a tautology
Is there a row in which X is true?
no
yes
X is a contradiction
X is contingent

Find truth tables for the formulas

- $P \wedge Q$
- $\neg(P \wedge Q)$
- $\neg P \vee \neg Q$
- $\neg P \wedge \neg Q$

	P	Q	$(P \wedge Q)$	$\neg(P \wedge Q)$	$(\neg P \vee \neg Q)$	$(\neg P \wedge \neg Q)$
1.	T	T	T	F	F	F
2.	T	F	F	T	T	F
3.	F	T	F	T	T	F
4.	F	F	F	T	T	T

$(P \wedge Q)$ and $\neg(P \wedge Q)$ are **contradictory**: they always have opposite truth values

	P	Q	$(P \wedge Q)$	$\neg(P \wedge Q)$	$(\neg P \vee \neg Q)$	$(\neg P \wedge \neg Q)$
1.	T	T	T	F	F	F
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	P	Q	$(P \wedge Q)$	$\neg(P \wedge Q)$	$(\neg P \vee \neg Q)$	$(\neg P \wedge \neg Q)$
1.	T	T	T	F	F	F
2.	T	F	F	T	T	F
3.	F	T	F	T	T	F
4.	F	F	F	T	T	T

$\neg(P \wedge Q)$ are $(\neg P \vee \neg Q)$ are **tautologically equivalent**: they always have the same truth values

	P	Q	$(P \wedge Q)$	$\neg(P \wedge Q)$	$(\neg P \vee \neg Q)$	$(\neg P \wedge \neg Q)$
1.	T	T	T	F	F	F
2.	T	F	F	T	T	F
3.	F	T	F	T	T	F
4.	F	F	F	T	T	T

$(P \wedge Q)$ are $(\neg P \wedge \neg Q)$ are **mutually exclusive**: they are never true in the same situation, but may be false in the same situation

	P	Q	$(P \wedge Q)$	$\neg(P \wedge Q)$	$(\neg P \vee \neg Q)$	$(\neg P \wedge \neg Q)$
1.	T	T	T	F	F	F
2.	T	F	F	T	T	F
3.	F	T	F	T	T	F
4.	F	F	F	T	T	T

$(\neg P \vee \neg Q)$ are $(\neg P \wedge \neg Q)$ are **satisfiable**: there is at least on situation in which both are true.

Classifying Two Formulas

Suppose that X and Y are two formulas.

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Suppose that X and Y are two formulas.

X and Y are **tautologically equivalent** provided that every truth-value assignment assigns the same truth value to X and Y.

X and Y are **contradictory** provided that every truth-value assignment assigns different truth values to X and Y.

X and Y are **mutually exclusive** provided that no truth-value assignment assigns true to both X and Y (though they may both be false).

X and Y are **satisfiable** provided that there is a truth-value assignment that assigns true to both X and Y.

Does every truth-value assignment assign the same truth value to X and Y?

If yes, then X and Y are tautologically equivalent.

Does every truth-value assignment assign different truth values to X and Y?

If yes, then X and Y are contradictory.

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If yes, then X and Y are tautologically equivalent.

Does every truth-value assignment assign different truth values to X and Y?

If yes, then X and Y are contradictory.

Is there some truth-value assignment that assigns T to both X and Y?

If yes, then X and Y are satisfiable.

If no, then X and Y are mutually exclusive.

Does every truth-value assignment assign the same truth value to X and Y?

If yes, then X and Y are tautologically equivalent.

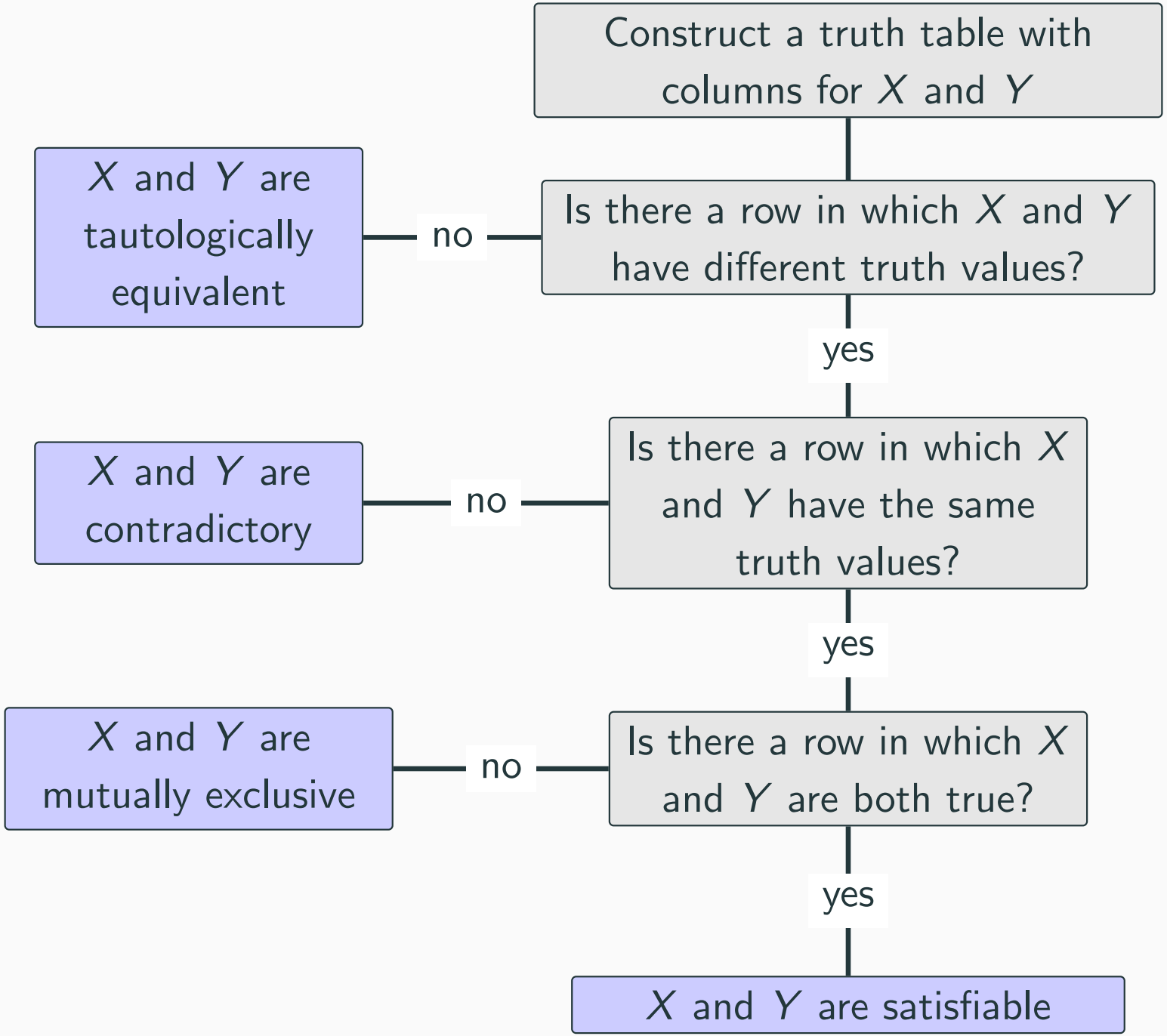
Does every truth-value assignment assign different truth values to X and Y?

If yes, then X and Y are contradictory.

Is there some truth-value assignment that assigns T to both X and Y?

If yes, then X and Y are satisfiable.

If no, then X and Y are mutually exclusive.


Construct a truth table with columns for X and Y
Is there a row in which X and Y have different truth values?
no
X and Y are tautologically equivalent
yes
Is there a row in which X and Y have the same truth values?
no
X and Y are contradictory
yes
Is there a row in which X and Y are both true?
no
X and Y are mutually exclusive
yes
X and Y are satisfiable