

Reasoning for Humans: Clear Thinking in an Uncertain World

PHIL 171

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Truth-Value Assignment

A **truth-value assignment** specifies a unique truth-value (either T or F) for each atomic formula.

Consider the formula $(A \rightarrow (A \vee B))$.

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The atomic subformulas are A and B

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There are 4 truth-value assignments for this formula:

1. A is T, B is T
2. A is T, B is F
3. A is F, B is T
4. A is F, B is F

How many truth value assignments are there for a single atomic proposition A ?

How many truth value assignments are there for a single atomic proposition A ? **2**

How many truth value assignments are there for a single atomic proposition A ? **2**

How many truth value assignments are there for two atomic propositions A and B ?

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How many truth value assignments are there for two atomic propositions A and B ? **4**

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How many truth value assignments are there for three atomic propositions A , B , and C ?

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How many truth value assignments are there for three atomic propositions A , B , and C ? **8**

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How many truth value assignments are there for three atomic propositions A , B , and C ? **8**

How many truth value assignments are there for four atomic propositions A , B , C and D ?

How many truth value assignments are there for a single atomic proposition A ? **2**

How many truth value assignments are there for two atomic propositions A and B ? **4**

How many truth value assignments are there for three atomic propositions A , B , and C ? **8**

How many truth value assignments are there for four atomic propositions A , B , C and D ? **16**

How many truth value assignments are there for a single atomic proposition A ? **2**

How many truth value assignments are there for two atomic propositions A and B ? **4**

How many truth value assignments are there for three atomic propositions A , B , and C ? **8**

How many truth value assignments are there for four atomic propositions A , B , C and D ? **16**

How many truth value assignments are there for n atomic propositions A_1, A_2, \dots, A_n ?

How many truth value assignments are there for a single atomic proposition A ? **2**

How many truth value assignments are there for two atomic propositions A and B ? **4**

How many truth value assignments are there for three atomic propositions A , B , and C ? **8**

How many truth value assignments are there for four atomic propositions A , B , C and D ? **16**

How many truth value assignments are there for n atomic propositions A_1, A_2, \dots, A_n ? 2^n

Truth Assignments

Given a truth assignment for all the atomic propositions in φ , how do we determine the truth value of φ ?

Conjunction

Eric had steak and wine. $(S \wedge W)$

$S \wedge W$



Conjunction

Eric had steak and wine. $(S \wedge W)$

S	W
T	T
T	F
F	T
F	F

Conjunction

Eric had steak and wine. $(S \wedge W)$

S	W	$(S \wedge W)$
T	T	T
T	F	F
F	T	F
F	F	F

Conjunction

Eric had steak and wine. $(S \wedge W)$

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S	W	$(S \wedge W)$
T	T	T
T	F	F
F	T	F
F	F	F

Truth-Table for Conjunction

φ	ψ	$(\varphi \wedge \psi)$
T	T	T
T	F	F
F	T	F
F	F	F

Disjunction

Eric had steak or wine. $(S \vee W)$

S	W	$(S \vee W)$
T	T	T
T	F	T
F	T	T
F	F	F

Disjunction

Eric had steak or wine. $(S \vee W)$

S	W	$(S \vee W)$
T	T	T
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Disjunction

Eric had steak or wine. $(S \vee W)$

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Disjunction

Eric had steak or wine. $(S \vee W)$

S	W	$(S \vee W)$
T	T	T
T	F	T
F	T	T
F	F	F

Disjunction

Eric had steak or wine. $(S \vee W)$

S	W	$(S \vee W)$
T	T	T
T	F	T
F	T	T
F	F	F

Truth-Table for Disjunction

φ	ψ	$(\varphi \vee \psi)$
T	T	T
T	F	T
F	T	T
F	F	F

Negation

Eric didn't have steak. $\neg S$

S	$\neg S$
T	F
F	T

Negation

Eric didn't have steak. $\neg S$

S	$\neg S$
T	F
F	T

Negation

Eric didn't have steak. $\neg S$

S	$\neg S$
T	F
F	T

Truth-Table for Negation

φ	$\neg\varphi$
T	F
F	T

$$P \wedge \neg(Q \vee R) \quad P \wedge \neg(Q \vee R)$$



$$P \wedge \neg(Q \vee R) \quad T$$

$$P \quad T$$

$$\neg(Q \vee R) \quad T$$

$$Q \vee R \quad \neg(Q \vee R)$$



$$\neg Q \quad \neg R \quad Q \vee R$$



$$Q \vee R \quad F$$

$$Q \quad F$$

$$R \quad F$$

$P \wedge \neg(Q \vee R) \quad P \wedge \neg(Q \vee R)$



$P \wedge \neg(Q \vee R) \quad \text{T}$

$P \quad \text{T}$

$\neg(Q \vee R) \quad \text{T}$

$Q \vee R \quad \neg(Q \vee R)$



$Q \vee R \quad \text{F}$

$Q \quad R \quad Q \vee R$



$Q \quad \text{F}$

$R \quad \text{F}$

$P \wedge \neg(Q \vee R) \quad P \wedge \neg(Q \vee R)$



$P \wedge \neg(Q \vee R) \quad \text{T}$

$P \quad \text{T}$

$\neg(Q \vee R) \quad \text{T}$

$Q \vee R \quad \neg(Q \vee R)$



$Q \vee R \quad \text{F}$

$Q \quad R \quad Q \vee R$



$Q \quad \text{F}$

$R \quad \text{F}$

$$P \wedge \neg(Q \vee R) \quad P \wedge \neg(Q \vee R)$$



$$P \wedge \neg(Q \vee R) \quad \text{T}$$

$$P \quad \text{T}$$

$$\neg(Q \vee R) \quad \text{T}$$

$$Q \vee R \quad \neg(Q \vee R)$$



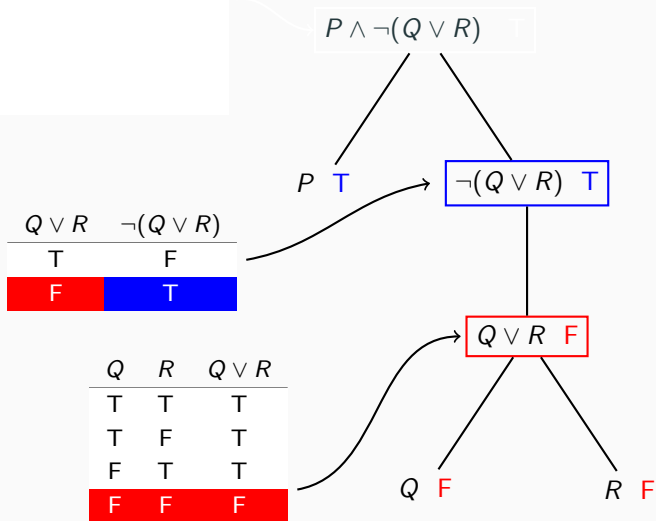
Q	R	Q \vee R
T	T	T
T	F	T
F	T	T
F	F	F

$$Q \vee R \quad \text{F}$$

$$Q \quad \text{F}$$

$$R \quad \text{F}$$

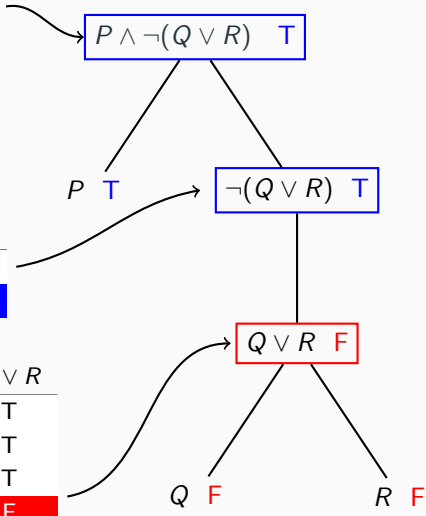
$$P \wedge \neg(Q \vee R) \quad P \wedge \neg(Q \vee R)$$



P	$\neg(Q \vee R)$	$P \wedge \neg(Q \vee R)$
T	T	T
T	F	F
F	T	F
F	F	F

$Q \vee R$	$\neg(Q \vee R)$
T	F
F	T

Q	R	$Q \vee R$
T	T	T
T	F	T
F	T	T
F	F	F



Recap: Truth Tables

φ	ψ	$(\varphi \wedge \psi)$
T	T	T
T	F	F
F	T	F
F	F	F

φ	ψ	$(\varphi \vee \psi)$
T	T	T
T	F	T
F	T	T
F	F	F

φ	$\neg\varphi$
T	F
F	T

Find truth tables for the formulas

- $P \wedge Q$
- $\neg(P \wedge Q)$
- $\neg P \vee \neg Q$
- $\neg P \wedge \neg Q$

P	Q	$(P \wedge Q)$	$\neg(P \wedge Q)$	$(\neg P \vee \neg Q)$	$(\neg P \wedge \neg Q)$
T	T	T	F	F	F
T	F	F	T	T	F
F	T	F	T	T	F
F	F	F	T	T	T

$(P \wedge Q)$ and $\neg(P \wedge Q)$ are contradictory: they always have opposite truth values

Material Conditional

If Eric had steak, then he had wine. $(S \rightarrow W)$

S	W	$(S \rightarrow W)$
T	T	T
T	F	F
F	T	T
F	F	T

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If Eric had steak, then he had wine. $(S \rightarrow W)$

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F	F	T

Material Conditional

If Eric had steak, then he had wine. $(S \rightarrow W)$

S	W	$(S \rightarrow W)$
T	T	T
T	F	F
F	T	T
F	F	T

Truth-Table for the Conditional

φ	ψ	$(\varphi \rightarrow \psi)$
T	T	T
T	F	F
F	T	T
F	F	T

Recap: Truth Tables

φ	ψ	$(\varphi \wedge \psi)$	φ	ψ	$(\varphi \vee \psi)$
T	T	T	T	T	T
T	F	F	T	F	T
F	T	F	F	T	T
F	F	F	F	F	F

φ	ψ	$(\varphi \rightarrow \psi)$
T	T	T
T	F	F
F	T	T
F	F	T

φ	$\neg\varphi$
T	F
F	T

A truth table for a formula φ is a table, where each row is a truth assignment for the atomic propositions in φ and there is a column for φ (and possible subformulas of φ) list the truth values of φ for each truth assignment.