Reasoning for Humans: Clear Thinking in an Uncertain World

PHIL 171

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An argument is **valid** if there is no situation in which all the premises are true and the conclusion is false.

If an argument is valid...

1. ...and the conclusion is false, then **at least one** of the premises must be false.
2. ...and **all the premises** are true, then the conclusion must be true.
3. ...then there are no counterexamples

Formulas

1. Every atomic formula $A, B, \ldots, P, Q, \ldots$ is a formula of sentential logic.

2. If φ is a formula of sentential logic, then so is $\neg\varphi$.

3. If φ and ψ are formulas of sentential logic, then so are each of the following:

   a. $(\varphi \wedge \psi)$
   b. $(\varphi \vee \psi)$
   c. $(\varphi \rightarrow \psi)$

4. An expression of sentential logic is a formula only if it can be constructed by one or more applications of the first three rules.

disjunction

$$(\varphi \lor \psi)$$

left disjunct

right disjunct

conjunction

$$(\varphi \wedge \psi)$$

left conjunct

right conjunct

implication/implies/conditional

$$(\varphi \rightarrow \psi)$$

antecedent

consequent

Procedure for Reinserting Omitted Parentheses

1. The outermost parentheses can be removed.

   For example, $((P \wedge R) \to Q)$ can be replaced with $(P \wedge R) \to Q$.

2. Group conjunctions and disjunctions before conditionals.

   For example, $(P \wedge Q) \to (R \vee S)$ is the same as $P \wedge Q \to R \vee S$.

3. Group conjunctions and disjunctions to the left before the ones on the right.

   For example, $(P \wedge Q) \wedge R$ is the same as $P \wedge Q \wedge R$.

4. Group conjunctions before disjunctions.

   For example, $P \vee (Q \wedge R)$ is the same as $P \vee Q \wedge R$.

Examples

1. $P \to Q \land \neg R \land P$

2. $P \to \neg\neg Q \land R$

3. $\neg P \land \neg(Q \lor R \to S)$

Examples

1. $P \to Q \land \neg R \land P$

   $(P \to ((Q \land \neg R) \land P))$

2. $P \to \neg\neg Q \land R$

   $(P \to (\neg\neg Q \land R))$

3. $\neg P \land \neg(Q \lor R \to S)$

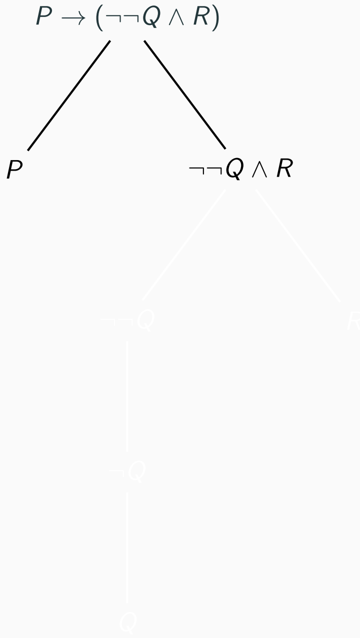
   $(\neg P \land \neg((Q \lor R) \to S))$

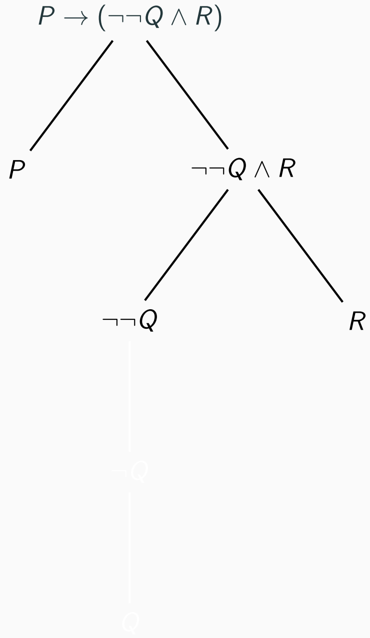
How do you find the syntax tree of a formula?

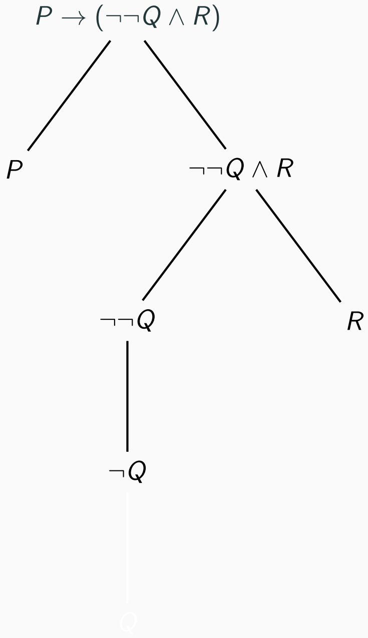
1. Write down the formula (adding parentheses if necessary).
2. Identify the main connective.
3. If it is a negation, write the negated formula below the current formula.
4. If it is a disjunction/conjunction/conditional, write the left disjunct/left conjunct/antecedent down left of the current formula and the right disjunct/right conjunct/consequent below right of the current formula.
5. Repeat steps 2-4 for for every formula that is not an atomic formula.
6. Draw lines connecting formula to the ones immediately below them.

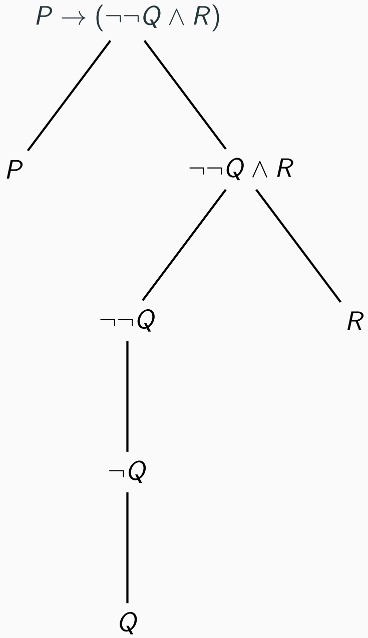
$P \to \neg\neg Q \land R$


$P \to (\neg\neg Q \land R)$
P
$\neg\neg Q \land R$
$\neg\neg Q$
R
$\neg Q$
Q


$P \to (\neg\neg Q \land R)$
P
$\neg\neg Q \land R$
$\neg\neg Q$
R
$\neg Q$
Q


P → (¬¬Q ∧ R)
P
¬¬Q ∧ R
¬¬Q
R


$P \to (\neg\neg Q \wedge R)$
P
$\neg\neg Q \wedge R$
$\neg\neg Q$
R
$\neg Q$


P → (¬¬Q ∧ R)
P
¬¬Q ∧ R
¬¬Q
R
¬Q
Q

Translating from English to formulas...

Simple Translations

1. John ran.
2. Mary laughed.
3. Harry said that Mary laughed.
4. John thinks that Mary laughed at his running.
5. John ran and Mary laughed.
6. Either John ran, or Mary laughed.
7. If Mary laughed, then John ran.
8. John didn't run.
9. It is not the case that Mary laughed.

Simple Translations

1. John ran. J
2. Mary laughed. M
3. Harry said that Mary laughed. H
4. John thinks that Mary laughed at his running. T
5. John ran and Mary laughed.
6. Either John ran, or Mary laughed.
7. If Mary laughed, then John ran.
8. John didn't run.
9. It is not the case that Mary laughed.

Simple Translations

1. John ran. J
2. Mary laughed. M
3. Harry said that Mary laughed. H
4. John thinks that Mary laughed at his running. T
5. John ran and Mary laughed. $J \land M$
6. Either John ran, or Mary laughed. $J \lor M$
7. If Mary laughed, then John ran. $M \to J$
8. John didn't run. $\neg J$
9. It is not the case that Mary laughed. $\neg M$

Inclusive vs. Exclusive “Or”

- Ann will have steak or Bob will have fish.
- Either Mary will buy ice cream, or John will buy ice cream.
- Ann will have steak or fish.
- Either Ann will get an A or a B in PHIL 171

Inclusive vs. Exclusive "Or"

- Ann will have steak or Bob will have fish.
- Either Mary will buy ice cream, or John will buy ice cream.
- Ann will have steak or fish.
- Either Ann will get an A or a B in PHIL 171

In a formula, '$\vee$' is always interpreted as an inclusive or.

Exclusive Disjunction

Eric will have steak or fish.

Exclusive Disjunction

Eric will have steak or fish.

Either Eric will have steak, or Eric will have fish (but not both).

Exclusive Disjunction

Eric will have steak or fish.

Either Eric will have steak, or Eric will have fish (but not both).

Either Eric will have steak, or Eric will have fish (but it is not the case that Eric will have steak and Eric will have fish).

Exclusive Disjunction

Eric will have steak or fish.

Either Eric will have steak, or Eric will have fish (but not both).

Either Eric will have steak, or Eric will have fish (but it is not the case that Eric will have steak and Eric will have fish).

$$(S \vee F) \wedge \neg(S \wedge F)$$

Conjunction Words other Than “And”

- The cat is napping, but the dog is chasing his tail.
- Although the cat is sharpening her claws on the dog, the dog is sleeping soundly.
- The cat is purring, though the dog is howling.
- Mary has just taken the dog to the vet; however, the dog’s appointment is tomorrow.
- Mary is fond of cats, whereas John likes dogs.

Conjunction Words other Than "And"

- The cat is napping, but the dog is chasing his tail. $(N \wedge C)$
- Although the cat is sharpening her claws on the dog, the dog is sleeping soundly. $(S \wedge D)$
- The cat is purring, though the dog is howling. $(P \wedge H)$
- Mary has just taken the dog to the vet; however, the dog's appointment is tomorrow. $(V \wedge A)$
- Mary is fond of cats, whereas John likes dogs. $(M \wedge J)$

Conditionals

- If John ran, then Mary laughed.
- If John ran, Mary laughed.
- Mary laughed, provided that John ran.
- Given that John ran, Mary laughed.
- Mary laughed if John ran.
- John ran only if Mary laughed.

Conditionals

- If John ran, then Mary laughed. ($J \rightarrow M$)
- If John ran, Mary laughed. ($J \rightarrow M$)
- Mary laughed, provided that John ran. ($J \rightarrow M$)
- Given that John ran, Mary laughed. ($J \rightarrow M$)
- Mary laughed if John ran. ($J \rightarrow M$)
- John ran only if Mary laughed. ($J \rightarrow M$)

Unless

John will pick up Henry at the airport, unless Mary does it.

Unless

John will pick up Henry at the airport, unless Mary does it.

If Mary doesn't pick up Henry at the airport, then John will pick up Henry at the airport.

Unless

John will pick up Henry at the airport, unless Mary does it.

If Mary doesn't pick up Henry at the airport, then John will pick up Henry at the airport.

$$\neg M \rightarrow J$$

Unless

John will pick up Henry at the airport, unless Mary does it.

If Mary doesn't pick up Henry at the airport, then John will pick up Henry at the airport.

$$\neg M \to J$$

$$M \vee J$$

$$\neg J \to M$$

Eric had steak and wine.

$(S \wedge W)$ $(W \wedge S)$

Eric had steak and wine.

✓ $(S \wedge W)$ ✗ $(W \wedge S)$

Eric had steak and wine.

✓ $(S \wedge W)$ ✗ $(W \wedge S)$

It's not the case that it is not raining.

$\neg\neg R$ R

Eric had steak and wine.

✓ $(S \wedge W)$ ✗ $(W \wedge S)$

It's not the case that it is not raining.

✓ $\neg\neg R$ ✗ R

Eric had steak and wine.

✓ $(S \wedge W)$ ✗ $(W \wedge S)$

It's not the case that it is not raining.

✓ $\neg\neg R$ ✗ R

Eric had neither red wine nor white wine for dinner.

$\neg(R \vee W)$ $(\neg R \wedge \neg W)$

Eric had steak and wine.

✓ $(S \wedge W)$ ✗ $(W \wedge S)$

It's not the case that it is not raining.

✓ $\neg\neg R$ ✗ R

Eric had neither red wine nor white wine for dinner.

✓ $\neg(R \vee W)$ ✓ $(\neg R \wedge \neg W)$

Eric had steak and wine.

✓ $(S \wedge W)$ ✗ $(W \wedge S)$

It's not the case that it is not raining.

✓ $\neg\neg R$ ✗ R

Eric had neither red wine nor white wine for dinner.

✓ $\neg(R \vee W)$ ✓ $(\neg R \wedge \neg W)$

Neither Eric nor Lauren had wine.

$\neg(E \vee L)$ $(\neg E \wedge \neg L)$

Eric had steak and wine.

✓ $(S \land W)$ ✗ $(W \land S)$

It's not the case that it is not raining.

✓ $\neg\neg R$ ✗ R

Eric had neither red wine nor white wine for dinner.

✓ $\neg(R \lor W)$ ✓ $(\neg R \land \neg W)$

Neither Eric nor Lauren had wine.

✓ $\neg(E \lor L)$ ✓ $(\neg E \land \neg L)$

Translation Example

A: Ann got an A in PHIL 171.
B: Bob got a A in PHIL 171.

Both Ann and Bob got an A in PHIL 171.

$A \wedge B$

Only Ann got an A in PHIL 171.

$A \wedge \neg B$

Translation Example

A: Ann got an A in PHIL 171.
B: Bob got a A in PHIL 171.

Both Ann and Bob got an A in PHIL 171.

$$A \wedge B$$

Only Ann got an A in PHIL 171.

$$A \wedge \neg B$$

Translation Example

A: Ann got an A in PHIL 171.
B: Bob got a A in PHIL 171.

At least one of Ann or Bob got an A in PHIL 171.

$$A \lor B$$

Exactly one of Ann or Bob got an A in PHIL 171.

$$(A \lor B) \land \neg(A \land B)$$

Translation Example

A: Ann got an A in PHIL 171.
B: Bob got a A in PHIL 171.

At least one of Ann or Bob got an A in PHIL 171.

$$A \vee B$$

Exactly one of Ann or Bob got an A in PHIL 171.

$$(A \vee B) \wedge \neg(A \wedge B)$$