

Reasoning for Humans: Clear Thinking in an Uncertain World

PHIL 171

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Translation Example

A: Ann got an A in PHIL 171.

B: Bob got a A in PHIL 171.

Both Ann and Bob got an A in PHIL 171.

$A \wedge B$

Only Ann got an A in PHIL 171.

$A \wedge \neg B$

Translation Example

A: Ann got an A in PHIL 171.

B: Bob got a A in PHIL 171.

Both Ann and Bob got an A in PHIL 171.

$$A \wedge B$$

Only Ann got an A in PHIL 171.

$$A \wedge \neg B$$

Translation Example

A: Ann got an A in PHIL 171.

B: Bob got a A in PHIL 171.

At least one of Ann or Bob got an A in PHIL 171.

$$A \vee B$$

Exactly one of Ann or Bob got an A in PHIL 171.

$$(A \vee B) \wedge \neg(A \wedge B)$$

$$(A \wedge \neg B) \vee (\neg A \wedge B)$$

Translation Example

A: Ann got an A in PHIL 171.

B: Bob got a A in PHIL 171.

At least one of Ann or Bob got an A in PHIL 171.

$$A \vee B$$

Exactly one of Ann or Bob got an A in PHIL 171.

$$(A \vee B) \wedge \neg(A \wedge B)$$

$$(A \wedge \neg B) \vee (\neg A \wedge B)$$

Translation Example

A: Ann got an A in PHIL 171.

B: Bob got an A in PHIL 171.

C: Carla got an A in PHIL 171.

At least one of Ann, Bob or Carla got an A in PHIL 171.

$$A \vee B \vee C$$

At least two of Ann, Bob, or Carla got an A in PHIL 171.

$$(A \wedge B \wedge \neg C) \vee (A \wedge \neg B \wedge C) \vee (\neg A \wedge B \wedge C) \vee (A \wedge B \wedge C)$$

Translation Example

A: Ann got an A in PHIL 171.

B: Bob got an A in PHIL 171.

C: Carla got an A in PHIL 171.

At least one of Ann, Bob or Carla got an A in PHIL 171.

$$A \vee B \vee C$$

At least two of Ann, Bob, or Carla got an A in PHIL 171.

$$(A \wedge B \wedge \neg C) \vee (A \wedge \neg B \wedge C) \vee (\neg A \wedge B \wedge C) \vee (A \wedge B \wedge C)$$

Truth-Value Assignment

A **truth-value assignment** specifies a unique truth-value (either T or F) for each atomic formula.

Consider the formula $(A \rightarrow (A \vee B))$.

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The atomic subformulas are A and B

Consider the formula $(A \rightarrow (A \vee B))$.

The atomic subformulas are A and B

There are 4 truth-value assignments for this formula:

1. A is T, B is T
2. A is T, B is F
3. A is F, B is T
4. A is F, B is F

Truth Assignments

Given a truth assignment for all the atomic propositions in X , how do we determine the truth value of X ?

Conjunction

Eric had steak and wine. $(S \wedge W)$

$S \wedge W$



Conjunction

Eric had steak and wine. $(S \wedge W)$

S	W
T	T
T	F
F	T
F	F

Conjunction

Eric had steak and wine. $(S \wedge W)$

S	W	$(S \wedge W)$
T	T	T
T	F	F
F	T	F
F	F	F

Conjunction

Eric had steak and wine. $(S \wedge W)$

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T	F	F
F	T	F
F	F	F

Conjunction

Eric had steak and wine. $(S \wedge W)$

S	W	$(S \wedge W)$
T	T	T
T	F	F
F	T	F
F	F	F

Truth-Table for Conjunction

X	Y	$(X \wedge Y)$
T	T	T
T	F	F
F	T	F
F	F	F

Disjunction

Eric had steak or wine. $(S \vee W)$

S	W	$(S \vee W)$
T	T	T
T	F	T
F	T	T
F	F	F

Disjunction

Eric had steak or wine. $(S \vee W)$

S	W	$(S \vee W)$
T	T	T
T	F	T
F	T	T
F	F	F

Disjunction

Eric had steak or wine. $(S \vee W)$

S	W	$(S \vee W)$
T	T	T
T	F	T
F	T	T
F	F	F

Disjunction

Eric had steak or wine. $(S \vee W)$

S	W	$(S \vee W)$
T	T	T
T	F	T
F	T	T
F	F	F

Disjunction

Eric had steak or wine. $(S \vee W)$

S	W	$(S \vee W)$
T	T	T
T	F	T
F	T	T
F	F	F

Truth-Table for Disjunction

X	Y	$(X \vee Y)$
T	T	T
T	F	T
F	T	T
F	F	F

Negation

Eric didn't have steak. $\neg S$

S	$\neg S$
T	F
F	T

Negation

Eric didn't have steak. $\neg S$

S	$\neg S$
T	F
F	T

Negation

Eric didn't have steak. $\neg S$

S	$\neg S$
T	F
F	T

Truth-Table for Negation

X	$\neg X$
T	F
F	T

$P \wedge \neg(Q \vee R) \quad P \wedge \neg(Q \vee R)$



$P \wedge \neg(Q \vee R) \quad T$

$P \quad T$

$\neg(Q \vee R) \quad T$

$Q \vee R \quad \neg(Q \vee R)$



$\neg Q \quad R \quad Q \vee R$



$Q \vee R \quad F$

$Q \quad F$

$R \quad F$

$P \wedge \neg(Q \vee R) \quad P \wedge \neg(Q \vee R)$



$P \wedge \neg(Q \vee R) \quad \text{T}$

$P \quad \text{T}$

$\neg(Q \vee R) \quad \text{T}$

$Q \vee R \quad \neg(Q \vee R)$



$Q \vee R \quad \text{F}$

$Q \quad R \quad Q \vee R$



$Q \quad \text{F}$

$R \quad \text{F}$

$P \wedge \neg(Q \vee R) \quad P \wedge \neg(Q \vee R)$



$P \wedge \neg(Q \vee R) \quad \text{T}$

$P \quad \text{T}$

$\neg(Q \vee R) \quad \text{T}$

$Q \vee R \quad \neg(Q \vee R)$



$Q \vee R \quad \text{F}$

$Q \quad R \quad Q \vee R$



$Q \quad \text{F}$

$R \quad \text{F}$

$$P \wedge \neg(Q \vee R) \quad P \wedge \neg(Q \vee R)$$



$$P \wedge \neg(Q \vee R) \quad \text{T}$$

$$P \quad \text{T}$$

$$\neg(Q \vee R) \quad \text{T}$$

$$Q \vee R \quad \neg(Q \vee R)$$



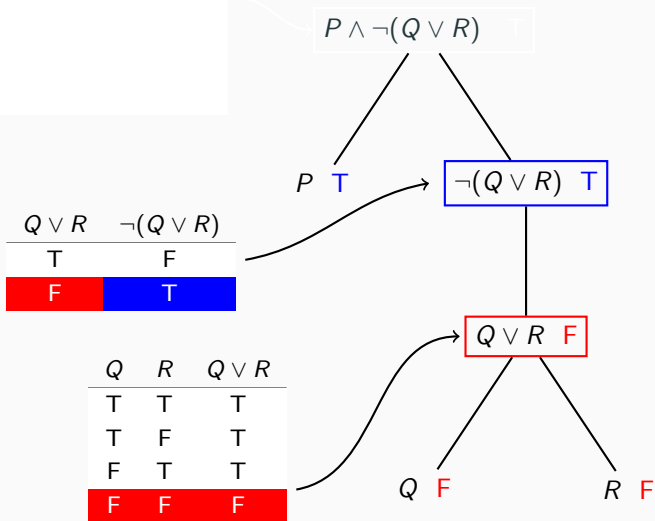
Q	R	Q ∨ R
T	T	T
T	F	T
F	T	T
F	F	F

$$Q \vee R \quad \text{F}$$

$$Q \quad \text{F}$$

$$R \quad \text{F}$$

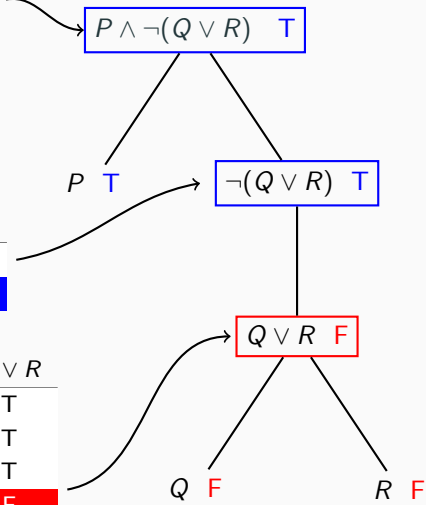
$$P \wedge \neg(Q \vee R) \quad P \wedge \neg(Q \vee R)$$



P	$\neg(Q \vee R)$	$P \wedge \neg(Q \vee R)$
T	T	T
T	F	F
F	T	F
F	F	F

$Q \vee R$	$\neg(Q \vee R)$
T	F
F	T

Q	R	$Q \vee R$
T	T	T
T	F	T
F	T	T
F	F	F



Recap: Truth Tables

X	Y	$(X \wedge Y)$
T	T	T
T	F	F
F	T	F
F	F	F

X	Y	$(X \vee Y)$
T	T	T
T	F	T
F	T	T
F	F	F

X	$\neg X$
T	F
F	T

Material Conditional

If Eric had steak, then he had wine. $(S \rightarrow W)$

S	W	$(S \rightarrow W)$
T	T	T
T	F	F
F	T	T
F	F	T

Material Conditional

If Eric had steak, then he had wine. $(S \rightarrow W)$

S	W	$(S \rightarrow W)$
T	T	T
T	F	F
F	T	T
F	F	T

Material Conditional

If Eric had steak, then he had wine. $(S \rightarrow W)$

S	W	$(S \rightarrow W)$
T	T	T
T	F	F
F	T	T
F	F	T

Material Conditional

If Eric had steak, then he had wine. $(S \rightarrow W)$

S	W	$(S \rightarrow W)$
T	T	T
T	F	F
F	T	T
F	F	T

Material Conditional

If Eric had steak, then he had wine. $(S \rightarrow W)$

S	W	$(S \rightarrow W)$
T	T	T
T	F	F
F	T	T
F	F	T

Truth-Table for the Conditional

X	Y	$(X \rightarrow Y)$
T	T	T
T	F	F
F	T	T
F	F	T

Recap: Truth Tables

X	Y	$(X \wedge Y)$	X	Y	$(X \vee Y)$
T	T	T	T	T	T
T	F	F	T	F	T
F	T	F	F	T	T
F	F	F	F	F	F

X	Y	$(X \rightarrow Y)$
T	T	T
T	F	F
F	T	T
F	F	T

X	$\neg X$
T	F
F	T

Given that P is T, Q is T and R is F, find the truth value of

$$P \rightarrow \neg(Q \wedge R)$$

P	$\neg(Q \wedge R)$	$P \rightarrow \neg(Q \wedge R)$
T	T	T
T	F	F
F	T	T
F	F	T

$Q \wedge R$	$\neg(Q \wedge R)$
T	F
F	T

Q	R	$Q \wedge R$
T	T	T
T	F	F
F	T	F
F	F	F

$P \rightarrow \neg(Q \wedge R)$ T

$\neg(Q \wedge R)$ T

$Q \wedge R$ F

Q T

R F

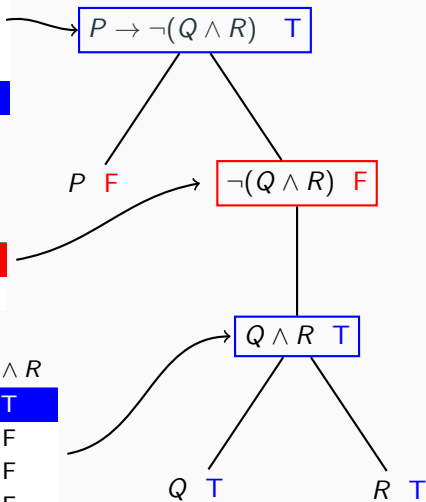
Given that P is F, Q is T and R is T, find the truth value of

$$P \rightarrow \neg(Q \wedge R)$$

P	$\neg(Q \wedge R)$	$P \rightarrow \neg(Q \wedge R)$
T	T	T
T	F	F
F	T	T
F	F	T

$Q \wedge R$	$\neg(Q \wedge R)$
T	F
F	T

Q	R	$Q \wedge R$
T	T	T
T	F	F
F	T	F
F	F	F



Given that P is T, Q is T and R is T, find the truth value of

$$P \rightarrow \neg(Q \wedge R)$$

P	$\neg(Q \wedge R)$	$P \rightarrow \neg(Q \wedge R)$
T	T	T
T	F	F
F	T	T
F	F	T

$Q \wedge R$	$\neg(Q \wedge R)$
T	F
F	T

Q	R	$Q \wedge R$
T	T	T
T	F	F
F	T	F
F	F	F

