Reasoning for Humans: Clear Thinking in an Uncertain World

PHIL 171

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Formulas

Formulas are constructed out of

1. Atomic propositions: Capital letters $A, B, C, \ldots$
2. Boolean connectives: $\wedge$, $\vee$, $\neg$, and $\rightarrow$
3. Parentheses:), (

Atomic Formulas

We use capital letters to represent **atomic formulas** or occasionally sentential letters: A, B, C, and so on (possibly with numeric subscripts).

Atomic Formauls

1. John ran.
2. Mary laughed.
3. Harry said that Mary laughed.
4. John thinks that Mary laughed at his running.
5. John ran and Mary laughed.
6. Either John ran, or Mary laughed.
7. If Mary laughed, then John ran.
8. John didn't run.
9. It is not the case that Mary laughed.

Atomic Formauls

1. John ran. R
2. Mary laughed. L
3. Harry said that Mary laughed. H
4. John thinks that Mary laughed at his running. M
5. John ran and Mary laughed.
6. Either John ran, or Mary laughed.
7. If Mary laughed, then John ran.
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Any capital letter can be used to represent an atomic proposition.

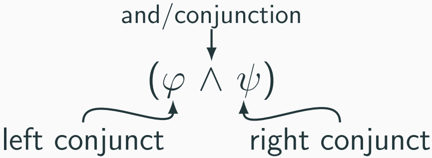
When translating from English to formulas, you must provide a translation key (association of a letter with an atomic sentence).

We use Greek letters (φ: "phi", ψ: "psi", α: "alpha", β: "beta", γ: "gamma", δ: "delta", etc.) as variables that range over all formulas.

E.g., In algebra, you write '$y = x + 2$'. In this expression, 'x' is a variable that can be assigned any number.

Conjunction

If φ and ψ are formulas, then $(\varphi \wedge \psi)$ is a formula, called a **conjunction**.



Conjunction

Ann had coffee and Bob had tea.

[Ann had coffee]$_1$ and [Bob had tea]$_2$.

C	Ann had coffee.
T	Bob had tea.

C and T

$C \wedge T$.

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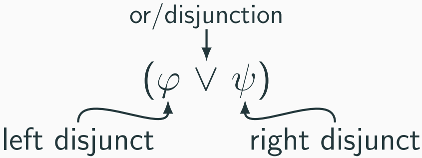
Disjunction

If φ and ψ are formulas, then $(\varphi \lor \psi)$ is a formula, called a **disjunction**.



Disjunction

Ann had coffee or Bob had tea.

[Ann had coffee]$_1$ or [Bob had tea]$_2$.

C	Ann had coffee.
T	Bob had tea.

C or T

$C \vee T$.

Disjunction

Ann had coffee or Bob had tea.

[Ann had coffee]$_1$ or [Bob had tea]$_2$.

C	Ann had coffee.
T	Bob had tea.

C or T

$C \wedge T$.

Disjunction

Ann had coffee or Bob had tea.

[Ann had coffee]$_1$ or [Bob had tea]$_2$.

C	Ann had coffee.
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C or T

$C \vee T$.

Disjunction

Ann had coffee or Bob had tea.

[Ann had coffee]$_1$ or [Bob had tea]$_2$.

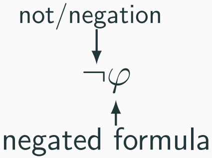
C	Ann had coffee.
T	Bob had tea.

C or T

$C \vee T$

Negation

If φ is a formula, then $(\neg\varphi)$ is a formula, called a **negation**.



Negation

Ann didn't have coffee.

It's not the case that Ann had coffee.

C	Ann had coffee.

It's not the case that [Ann had coffee]$_1$.

$\neg C$

Negation

Ann didn't have coffee.

It's not the case that Ann had coffee.

C	Ann had coffee.

It's not the case that [Ann had coffee]$_1$.

$\neg C$

Negation

Ann didn't have coffee.

It's not the case that Ann had coffee.

C	Ann had coffee.

It's not the case that [Ann had coffee]$_1$.

$\neg C$

Negation

Ann didn't have coffee.

It's not the case that Ann had coffee.

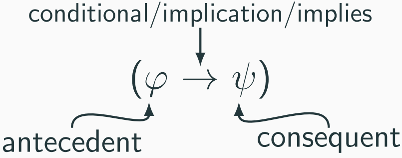
C	Ann had coffee.

It's not the case that [Ann had coffee]$_1$.

$\neg C$

Conditional

If φ and ψ are formulas, then $(\varphi \to \psi)$ is a formula, called a **conditional** (sometimes this is called the **material conditional**).



Conditional

If Ann had coffee, then Bob had tea.

Conditional

If Ann had coffee, then Bob had tea.

If [Ann had coffee]$_1$, then [Bob had tea]$_2$.

C	Ann had coffee.
T	Bob had tea.

C and T.

$C \wedge T$.

Conditional

If Ann had coffee, then Bob had tea.

If [Ann had coffee]$_1$, then [Bob had tea]$_2$.

C	Ann had coffee.
T	Bob had tea.

If C, then T.

$C \to T$.

Conditional

If Ann had coffee, then Bob had tea.

If [Ann had coffee]$_1$, then [Bob had tea]$_2$.

C	Ann had coffee.
T	Bob had tea.

If C, then T.

$C \rightarrow T$

Formulas

1. Every atomic formula is a formula of sentential logic.
2. If φ is a formula of sentential logic, then so is $\neg\varphi$.
3. If φ and ψ are formulas of sentential logic, then so are each of the following:
   a. $(\varphi \land \psi)$
   b. $(\varphi \lor \psi)$
   c. $(\varphi \to \psi)$
4. An expression of sentential logic is a formula only if it can be constructed by one or more applications of the first three rules.

Why is $(\neg(A \wedge \neg B) \rightarrow (C \vee \neg D))$ a formula?

1. Every atomic formula is a formula of sentential logic.

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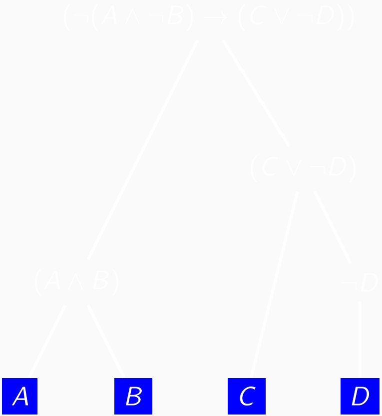

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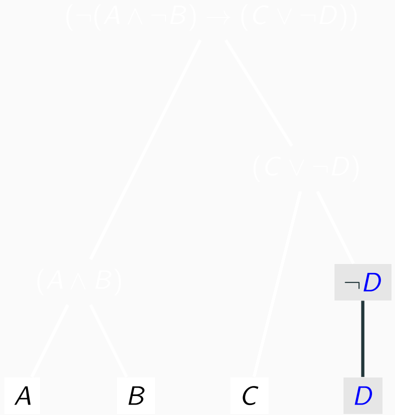

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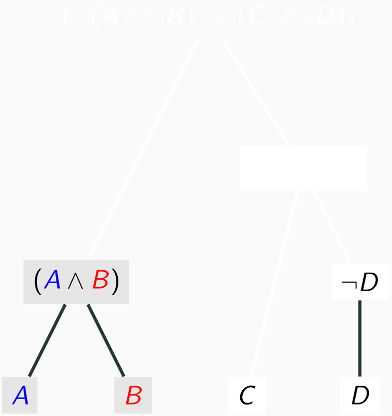

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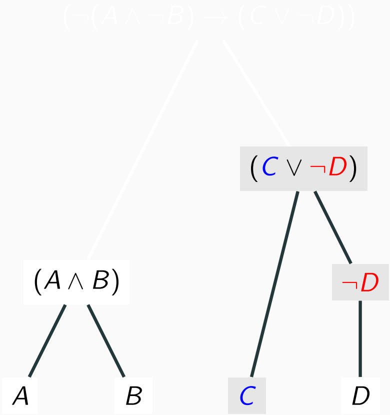

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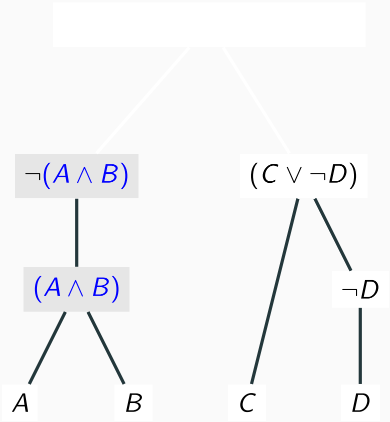

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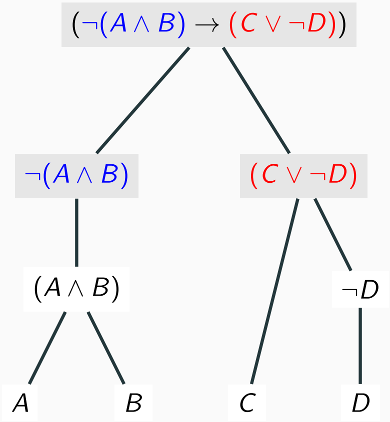

How do you find the syntax tree of a formula?

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What is the **main connective** of a formula?

He has an Ace if he does not have a Knight or a Spade.

$$(\neg(K \vee S) \rightarrow A)$$

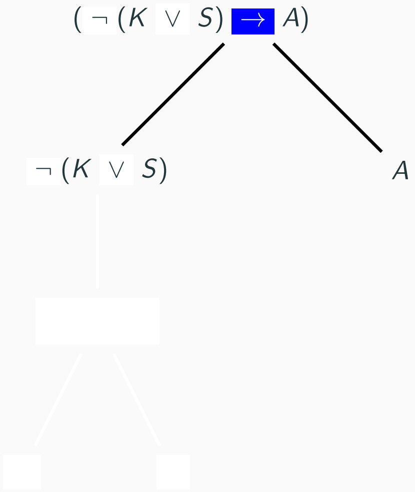
Syntax Tree

$$(\neg(K \vee S) \rightarrow A)$$

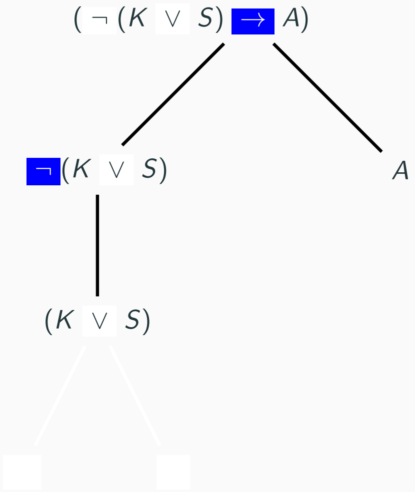
Syntax Tree

$$(\neg (K \lor S) \to A)$$

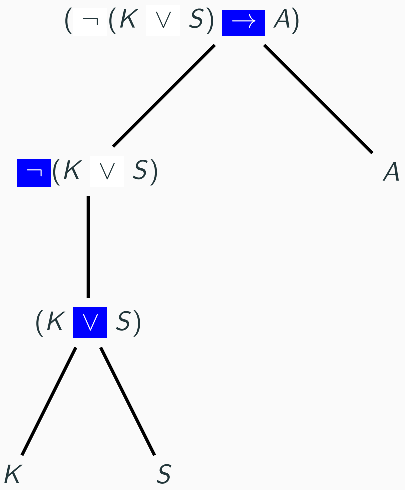
Syntax Tree



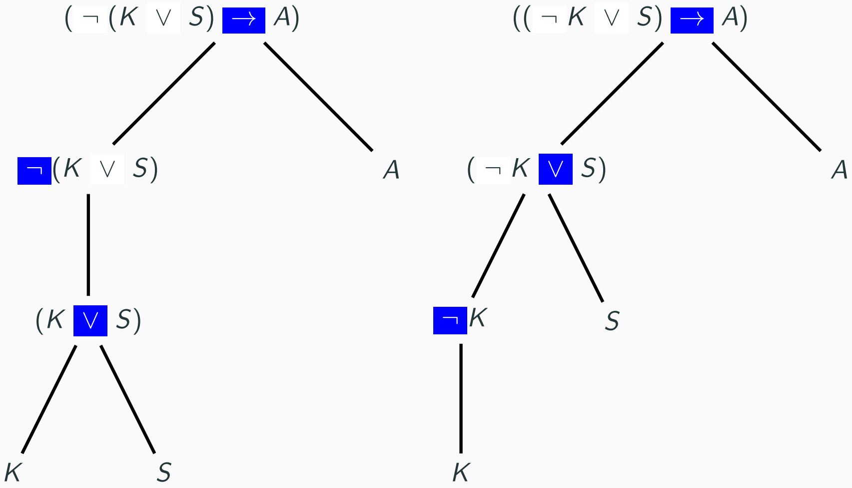
Syntax Tree



Syntax Tree



$(\neg(K \vee S) \rightarrow A)$ $((\neg K \vee S) \rightarrow A)$

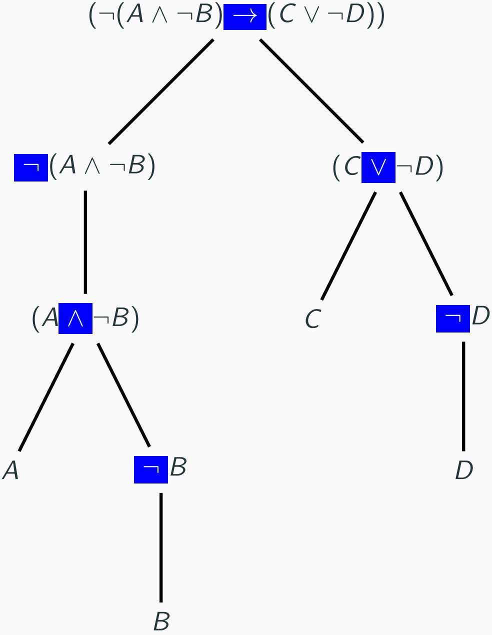


(¬(K ∨ S) → A)
¬(K ∨ S)
A
(K ∨ S)
K
S
((¬K ∨ S) → A)
(¬K ∨ S)
A
¬K
S
K

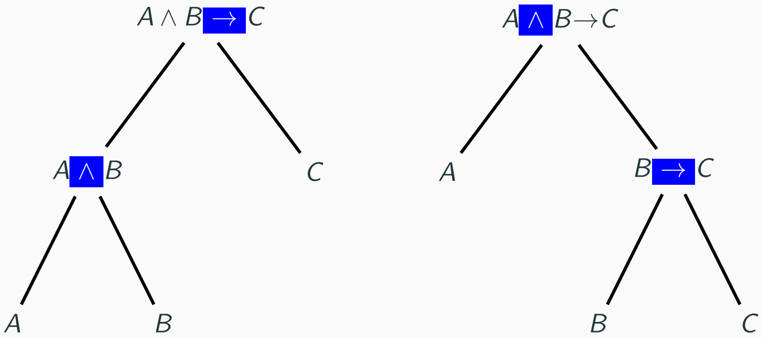
How do you find the syntax tree of a formula?

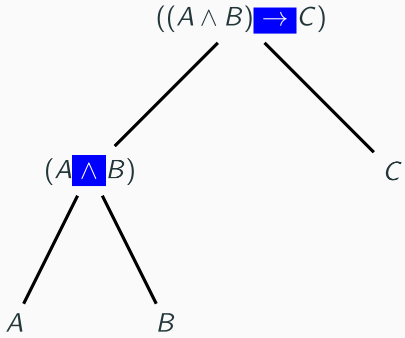
1. Write down the formula (adding parentheses if necessary).
2. Identify the main connective.
3. If it is a negation, write the negated formula below the current formula.
4. If it is a disjunction/conjunction/conditional, write the left disjunct/left conjunct/antecedent down left of the current formula and the right disjunct/right conjunct/consequent below right of the current formula.
5. Repeat steps 2-4 for for every formula that is not an atomic formula.
6. Draw lines connecting formula to the ones immediately below them.

Draw the syntax tree for $(\neg(A \wedge \neg B) \rightarrow (C \vee \neg D))$.


$(\neg(A \wedge \neg B) \rightarrow (C \vee \neg D))$
$\neg(A \wedge \neg B)$
$(C \vee \neg D)$
$(A \wedge \neg B)$
C
$\neg D$
A
$\neg B$
D
B

$$A \wedge B \rightarrow C$$


A ∧ B → C
A ∧ B
C
A
B
A ∧ B → C
A
B → C
B
C



$((A \wedge B) \rightarrow C)$
$(A \wedge B)$
C
A
B

Procedure for Reinserting Omitted Parentheses

1. The outermost parentheses can be removed.

   For example, $((P \wedge R) \rightarrow Q)$ can be replaced with $(P \wedge R) \rightarrow Q$.

2. Group conjunctions and disjunctions before conditionals.

   For example, $(P \wedge Q) \rightarrow (R \vee S)$ is the same as $P \wedge Q \rightarrow R \vee S$.

3. Group conjunctions and disjunctions to the left before the ones on the right.

   For example, $(P \wedge Q) \wedge R$ is the same as $P \wedge Q \wedge R$.

4. Group conjunctions before disjunctions.

   For example, $P \vee (Q \wedge R)$ is the same as $P \vee Q \wedge R$.

Add Parentheses

$$A \wedge B \wedge C$$

$$A \wedge B \vee C$$

$$\neg A \rightarrow C$$

$$\neg A \wedge B \rightarrow C$$

$$\neg A \wedge B \vee C$$

Add Parentheses

$$((A \wedge B) \wedge C)$$

$$((A \wedge B) \vee C)$$

$$(\neg A \rightarrow C)$$

$$((\neg A \wedge B) \rightarrow C)$$

$$((\neg A \wedge B) \vee C)$$