# Reasoning for Humans: Clear Thinking in an Uncertain World

PHIL 171

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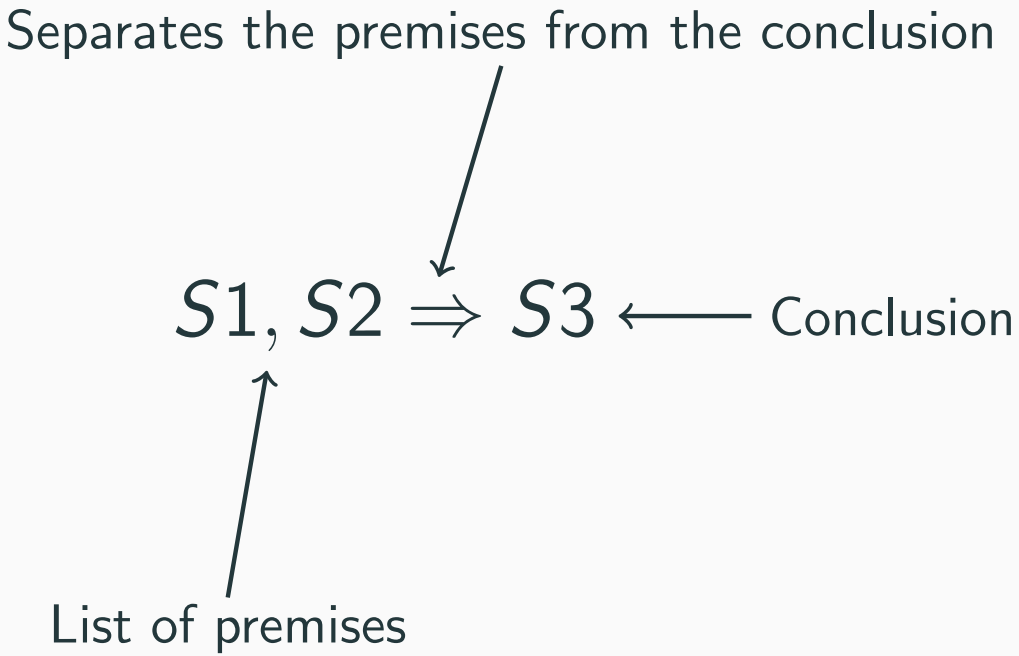
# Introductory Topics

- ✓ Arguments
- ✓ Declarative sentences, propositions
- ✓ Representing arguments: $P_1, P_2, P_3 \Rightarrow C$
- ✓ Argument form
- Valid arguments and inferences


Separates the premises from the conclusion
$S1, S2 \Rightarrow S3$
Conclusion
List of premises

Eric had steak **or** fish for dinner. Eric did **not** have fish. $\Rightarrow$ Eric had steak for dinner.

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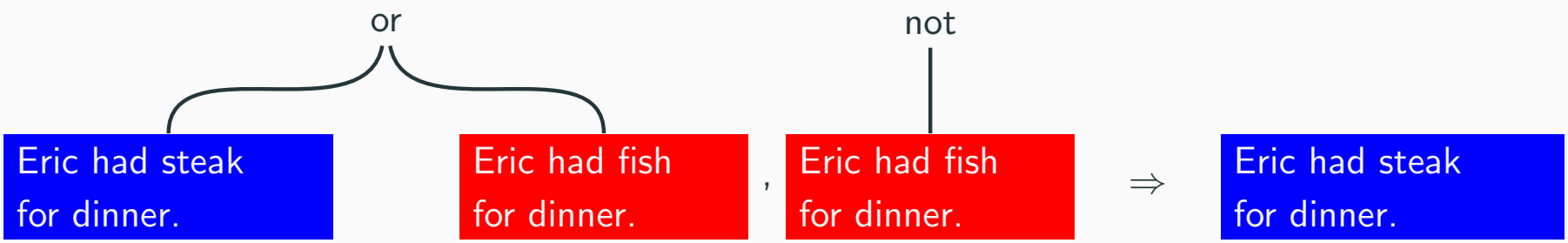


or
not
Eric had steak for dinner.
Eric had fish for dinner.
,
Eric had fish for dinner.
⇒
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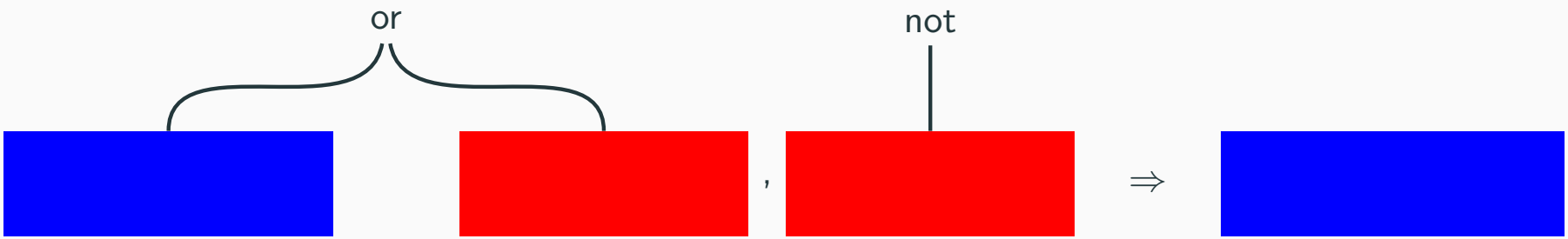
or
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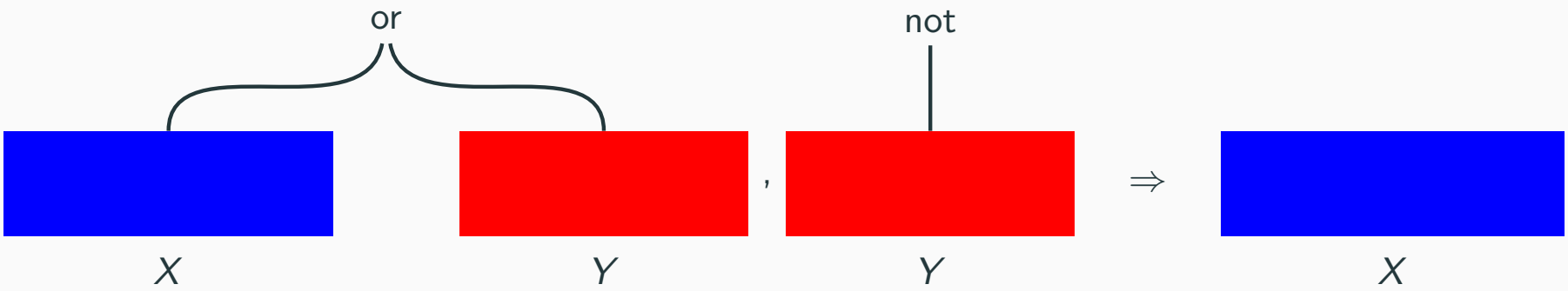
,
⇒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,
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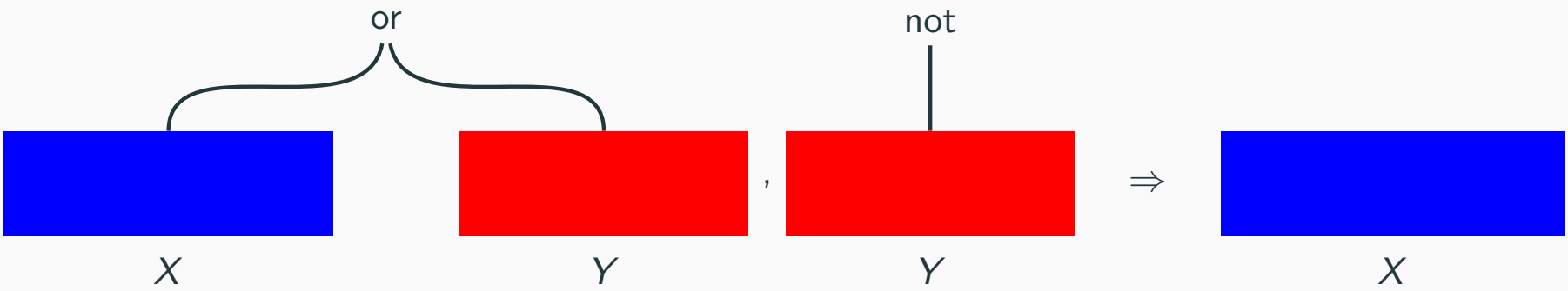
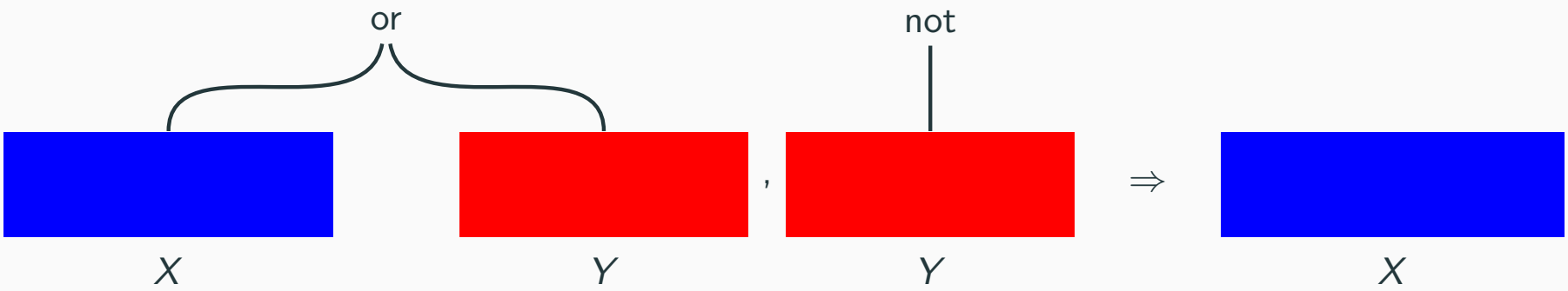
X
Y
Y
X

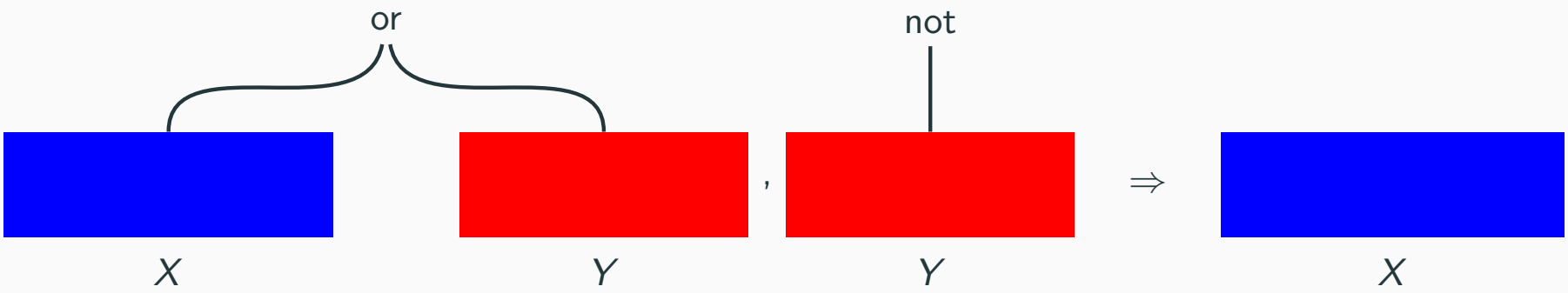



$X$ or $Y$, not $Y \Rightarrow X$




$X$ or $Y$, not $Y \Rightarrow X$

- Ann will get an A or B in PHIL 171. Ann will not get a B in PHIL 171. So, Ann will get an A in PHIL 171.
- My keys are in my office or in my car. My keys are not in my car. So, my keys are in my office.
- The lecture is in LeFrak or on Zoom. The lecture is not on Zoom. So, the lecture is in LeFrak.
- ...

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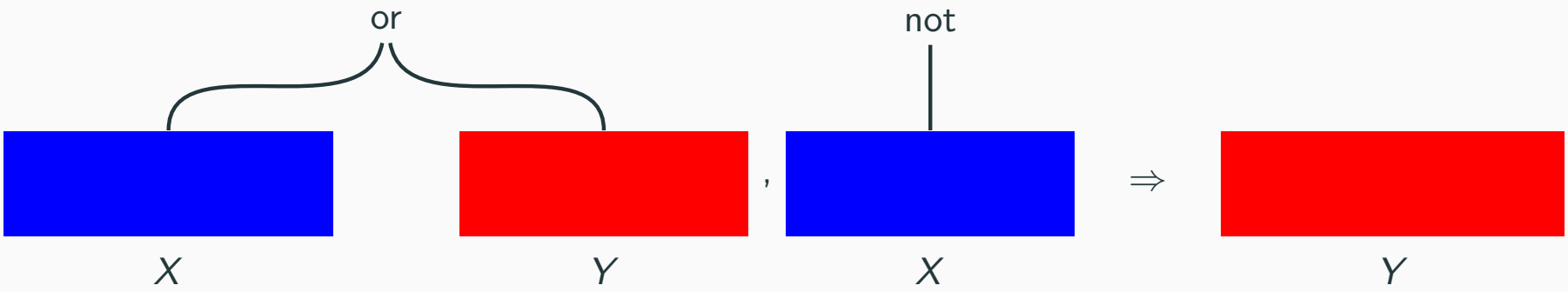
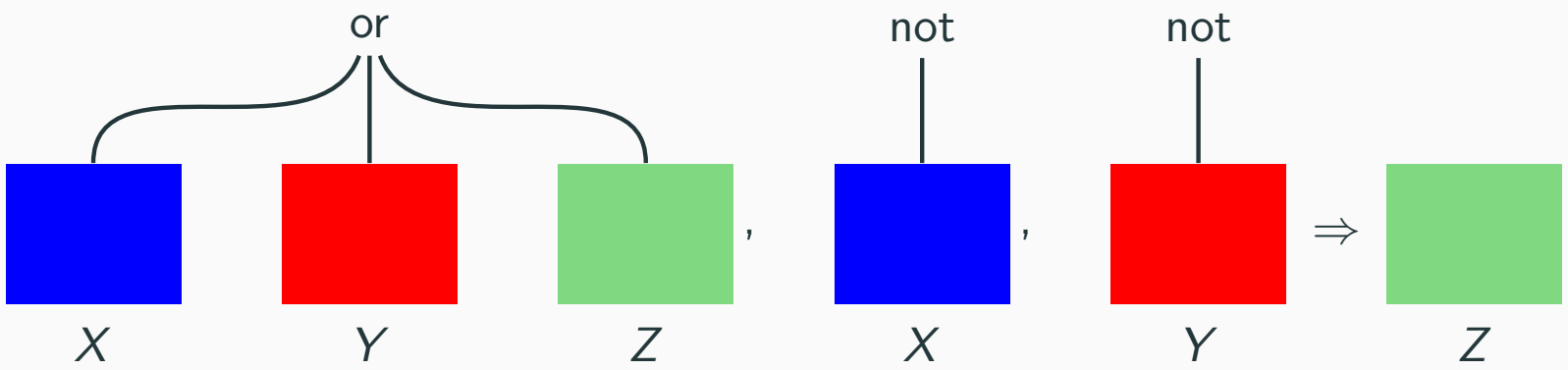



$X$ or $Y$, not $X \Rightarrow Y$




$X$ or $Y$ or $Z$, not $X$, not $Y \Rightarrow Z$

# Restaurant Example

In a restaurant, Ann ordered Fish, Bob ordered Pasta and Charles ordered Meat. Out of the kitchen comes some new person carrying the three plates. What will happen?

The waiter asks a first question, say "Who ordered the meat?", and puts that plate in front of Charles. Then he asks a second question "Who ordered the fish?", and puts that plate in front of Ann.

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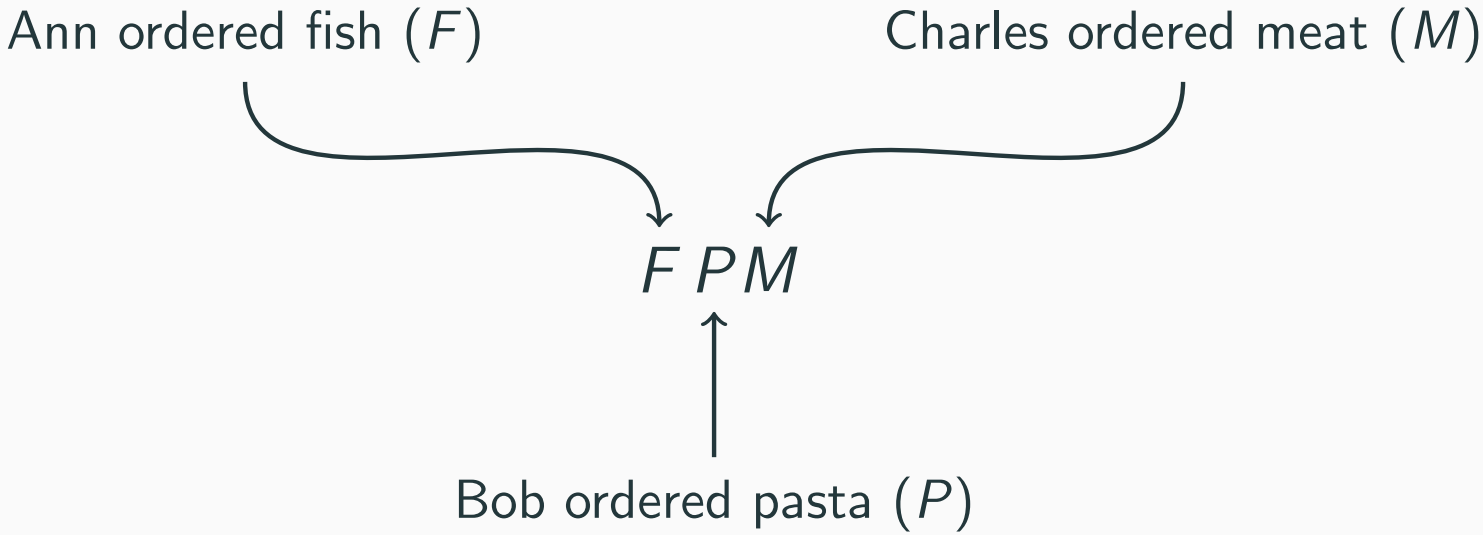
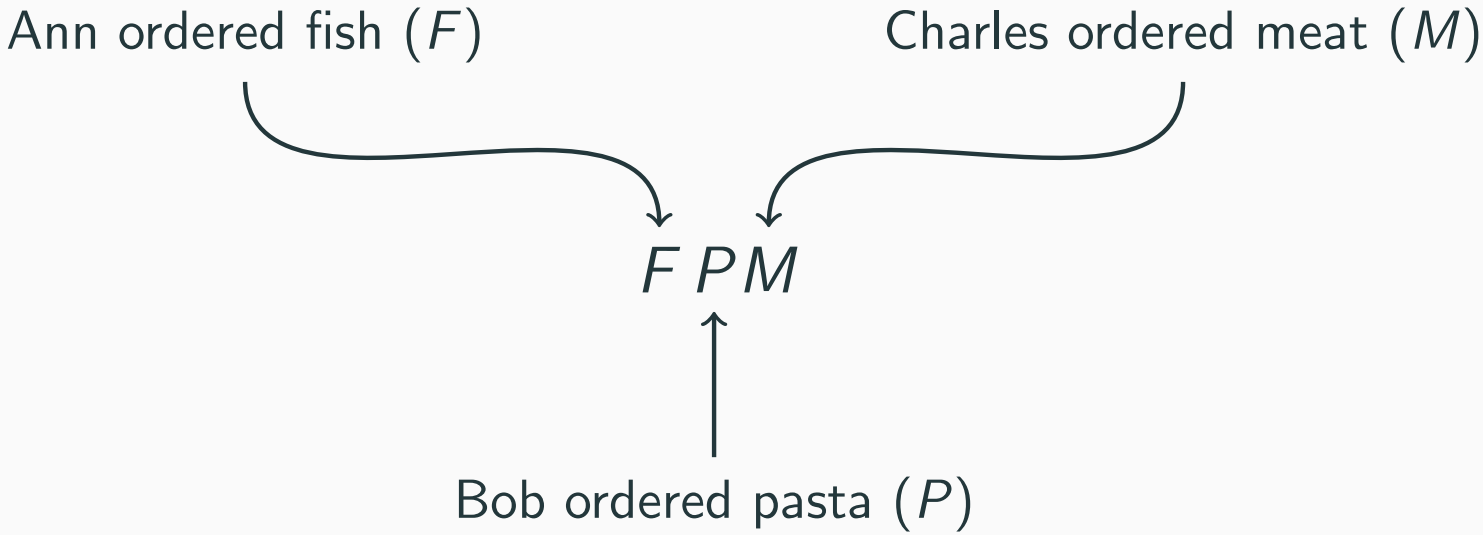
Meat or Pasta or Fish, not Fish, not Meat $\Longrightarrow$ Pasta




How many ways could the waiter/waitress distribute the meals?

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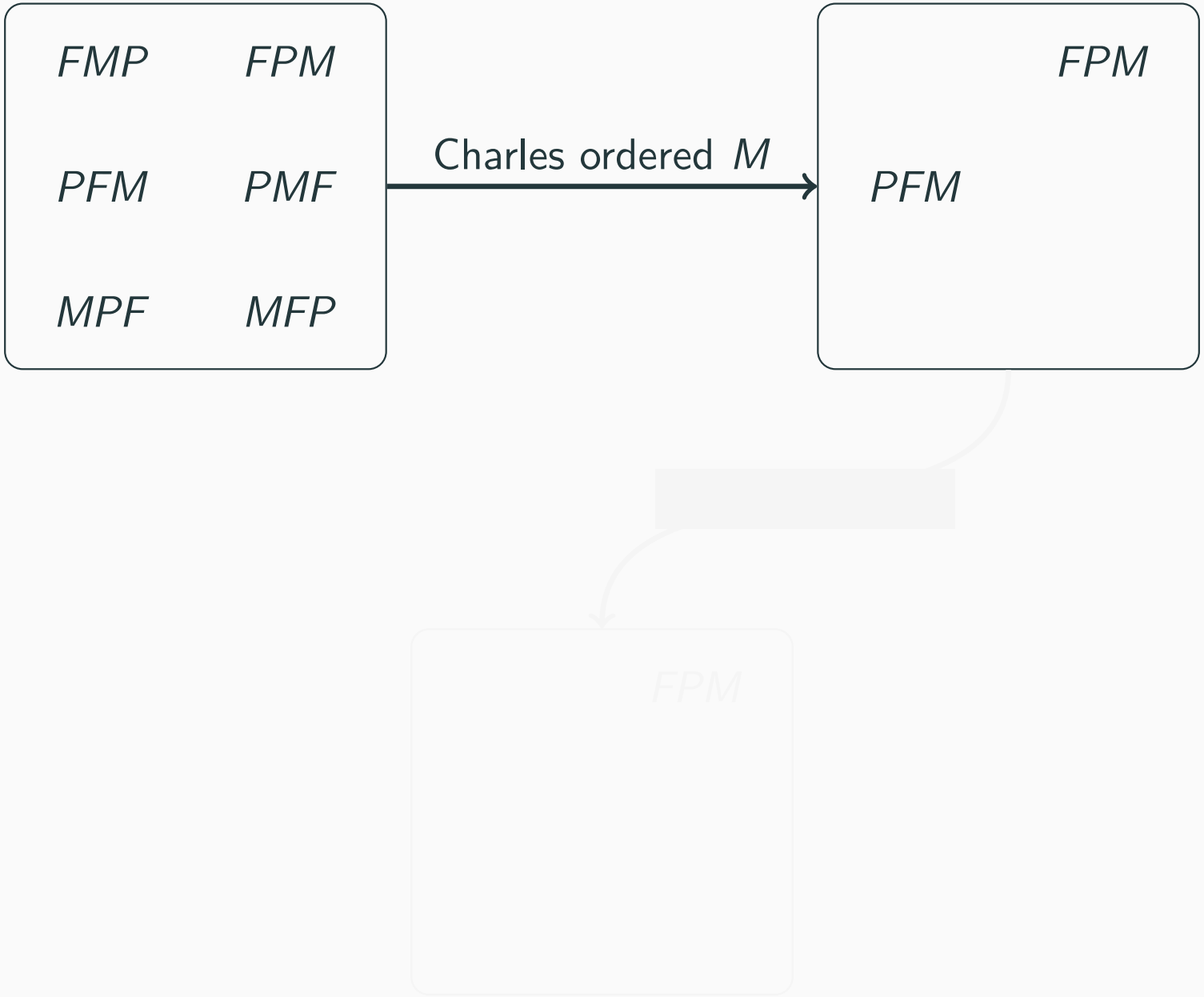
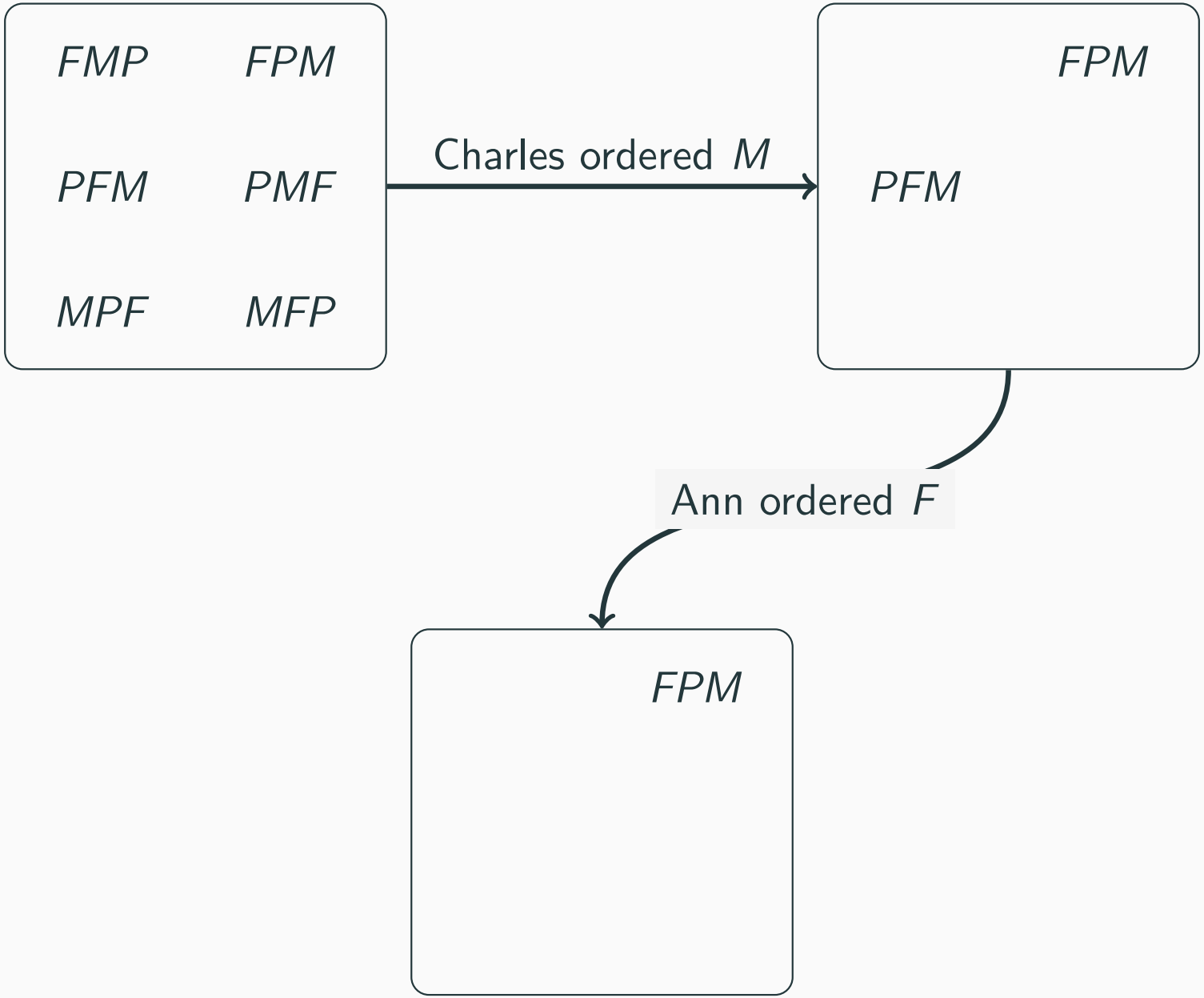

FMP
FPM
PFM
PMF
MPF
MFP
Charles ordered M
FPM
PFM
FPM


FMP
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PMF
MPF
MFP
Charles ordered M
FPM
PFM
Ann ordered F
FPM

After *observing/learning* that Charles ordered meat and Ann ordered fish, the waiter/waitress **concludes**/**infers** that Bob ordered pasta ($P$). That is, the only possibility is $FPM$.

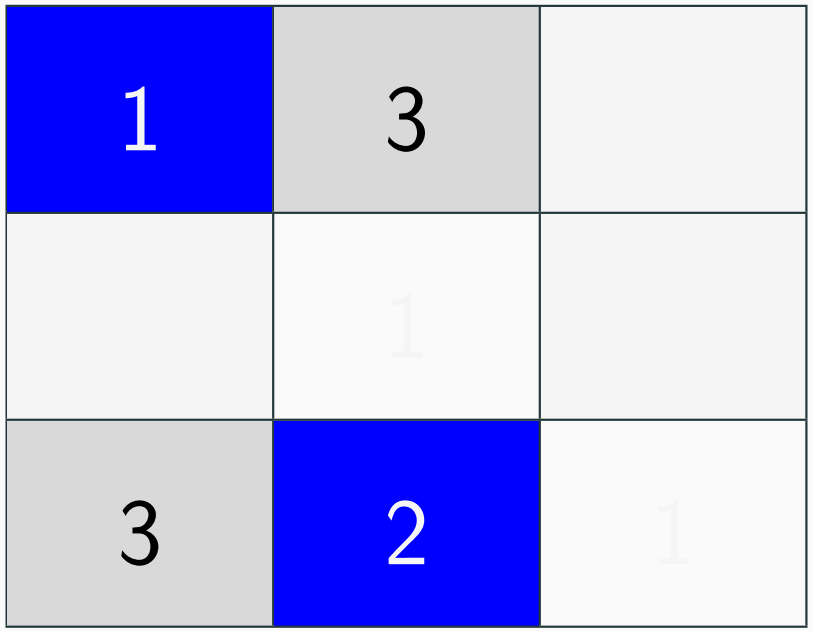
$M$ or $F$ or $P$, not $M$, not $F \Longrightarrow P$

# Sudoku

| | | |
|---|---|---|
| 1 | | |
| | 1 | |
| 3 | 2 | 1 |

1 or 2 or 3, not 1, not 2 $\Longrightarrow$ 3

# Sudoku

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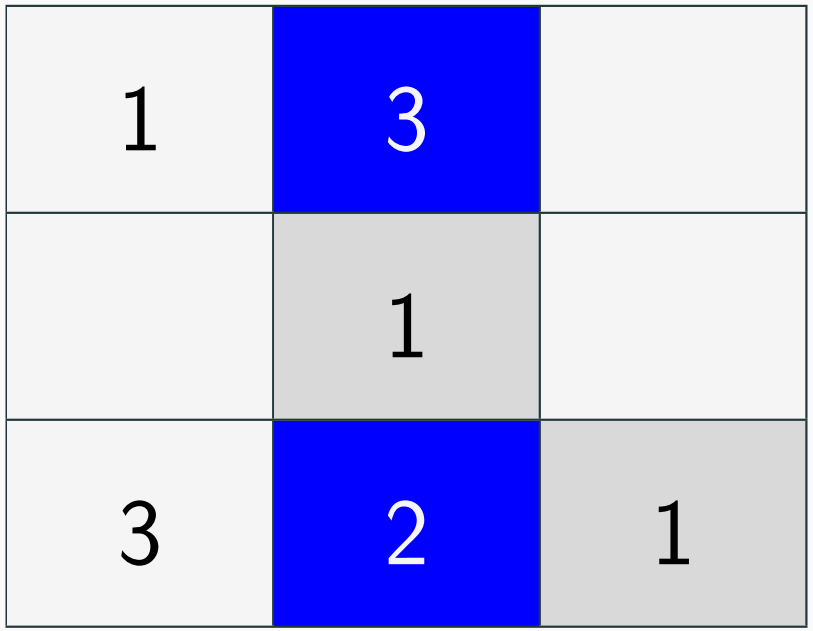
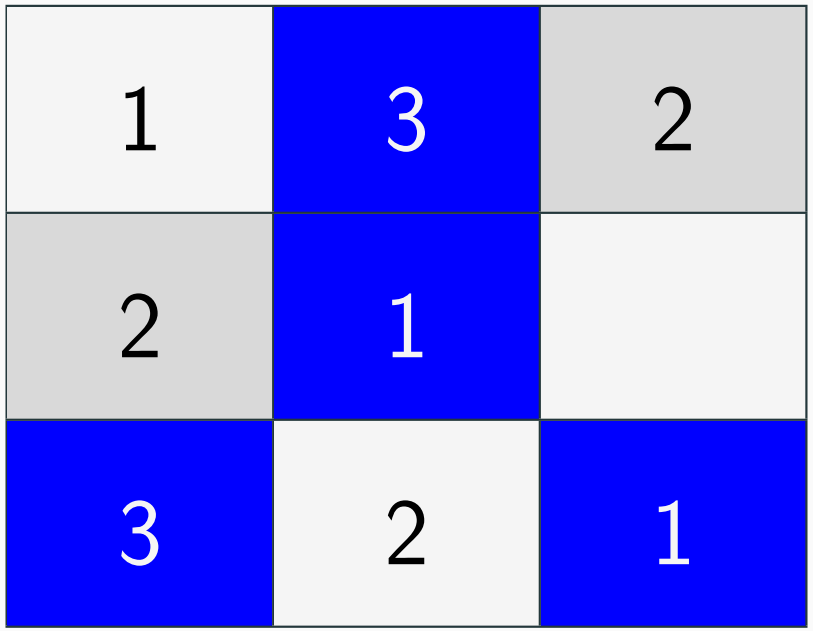



1 or 2 or 3, not 2, not 3 $\Longrightarrow$ 1

# Sudoku




1 or 2 or 3, not 3, not 1 $\Longrightarrow$ 2

# Sudoku

| | | |
|---|---|---|
| 1 | 3 | 2 |
| 2 | 1 | 3 |
| 3 | 2 | 1 |

1 or 2 or 3, not 2, not 1 $\Longrightarrow$ 3

# Argument form/inference pattern

**From** fish or meat or pasta, not fish, not meat **infer** pasta

**From** 1 or 2 or 3, not 1, not 2 **infer** 3

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$$X \text{ or } Y \text{ or } Z, \text{ not } X, \text{ not } Y \Rightarrow Z$$

# Important point

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Inferring is an activity that a person or computer performs, but "follows from" is a relationship between sentences.

# Valid Argument

An argument is **valid** if it has a form that makes it impossible for all of the premises to be true and the conclusion false.

# Variables

We use letters at the end of the alphabet ($X$, $Y$, $Z$) as **variables** for propositions.

This is analogous to the way we use variables to represent any number in algebra: e.g., $x + 2$

So, in the argument: $X$ or $Y$ or $Z$, not $X$, not $Y \Rightarrow Z$, each of $X$, $Y$ and $Z$ can be replaced by statements.

- Different variables may be replaced by the same statement
- The same variable may occur more than once in an an expression: You must replace that variable with the same statement.

(Note that the words "or" and "not" have a fixed interpretation.)

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- Eric held his breath for 15 minutes.
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- $2 + 2 = 5$.
  (arithmetic impossibility)
- Eric grew up in Ohio and Eric did not grow up in Ohio.
  (logical impossibility)

An argument is **valid** if it has a form that makes it **impossible** for all of the premises to be true and the conclusion false.

An argument is **sound** if it is valid and all the premises are true.

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# Propositional Logic

- Boolean Connectives
- Formulas
- Translation
- Truth of Formulas
- Truth Tables
- Classifying Formulas
- Valid Arguments

# Connectives

In any language, there are words or phrases that are used to construct complex statements out of simpler ones.

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In any language, there are words or phrases that are used to construct complex statements out of simpler ones.

For instance, consider the following two statements:

- It is cold.
- College Park is in Maryland.

The words in boldface transform these two statements into new, more complex statements:

- It is **not** cold.
- It is cold **or** College Park is in Maryland.
- It is cold **and** College Park is in Maryland.

# Examples of Connectives

| Term | Example sentence |
|---|---|
| but | It is raining, **but** I have an umbrella. |
| either...or | **Either** Ann will have chocolate **or** vanilla ice cream. |
| if | Ann will get an A in PHIL 171 **if** she asks questions in class. |
| only if | Ann will ask questions in class **only if** the material discussed in the class is interesting. |
| if...then | **If** it is raining, **then** I will bring an umbrella. |
| unless | I will pick up Lauren from the airport **unless** Angelie picks her up. |
| because | I got wet **because** it is raining. |

# Truth-Functional Connectives

A connective is **truth functional** when the truth or falsity of a complex statement constructed using the connective is completely determined by the truth or falsity of the statements to which the connective is applied.

# Non-Truth-Functional Connective: "Because"

"$P$ because $Q$" is true only if both $P$ and $Q$ are true. But, some instances of "$P$ because $Q$" are false, even though both $P$ and $Q$ are true.

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Let $P$ be the true claim that George W. Bush was president in 2001, and $Q$ be the true claim that it rained in College Park in 2020.

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Another example:

1. I am wet because it is raining.
2. It is raining because I am wet.

# Boolean Connectives

| English expression | Logical connective |
|---|---|
| not, it is not the case that, it is false that | $\neg$ |
| and, yet, but, however, both, also, although, nevertheless, still, also, although, moreover, additionally, furthermore | $\wedge$ |
| or, unless, either ... or ... | $\vee$ |
| if ... then ..., only if, given that, in case, provided that, on condition that, sufficient condition, necessary condition, unless | $\rightarrow$ |

For all statements $X$ and $Y$, we write:

- "$X \wedge Y$" instead of "$X$ and $Y$".
- "$X \vee Y$" instead of "$X$ or $Y$".
- "$\neg X$" instead of " not $X$".
- "$X \rightarrow Y$" instead of "if $X$ then $Y$".

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So, the abstract argument:

$$X \text{ or } Y, \text{ not } X \Rightarrow Y$$

will be written as:

$$X \vee Y, \neg X \Rightarrow Y.$$