Reasoning for Humans: Clear Thinking in an Uncertain World

PHIL 171

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Conceptions of Belief

Binary: "all-out" or "full" belief. For any proposition φ, either you believe φ, disbelieve φ (i.e., believe $\neg\varphi$), or suspend judgement about φ (neither believe φ nor believe $\neg\varphi$).

Graded: "credences", beliefs come in degrees. We are *more confident* in some of our beliefs than in others.

Eric Schwitzgebel. *Belief*. In The Stanford Encyclopedia of Philosophy.

Franz Huber. *Formal Theories of Belief*. In The Stanford Encyclopedia of Philosophy.

Constraints on Rational Beliefs/Credences

Let $Bel(\varphi)$ mean that φ is believed. Rational beliefs should satisfy the following principles:

- Consistency: If $Bel(\varphi)$, then φ is satisfiable.
- Aggregation: If $Bel(\varphi)$ and $Bel(\psi)$, then $Bel(\varphi \wedge \psi)$

Let $Cr(\varphi)$ be the credence that φ is true. Rational credences (graded beliefs) should satisfy the axioms of probability

- For any formula φ, $Cr(\neg\varphi) = 1 - Cr(\varphi)$
- If φ is a contradiction, then $Cr(\varphi) = 0$
- If φ and ψ are mutually exclusive, then $Cr(\varphi \vee \psi) = Cr(\varphi) + Cr(\psi)$

What is the relationship between full beliefs and credences?

D. Christensen. *Putting Logic in its Place*. Oxford University Press.

Bridge Principles

Probability 1: For all φ, $Bel(\varphi)$ if, and only if, $Cr(\varphi) = 1$

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The Lockean Thesis: There is some threshold $0.5 \leq t \leq 1$ such that for all φ, $Bel(\varphi)$ if, and only if, $Cr(\varphi) > t$

Credence, Beliefs, Chances

$Cr(\varphi)$ the credence that φ is true.

$Bel(\varphi)$ the belief that φ is true.

Chances: Consider beliefs/credences in propositions of the form "there is a chance c that φ is true". For example,

- there is a 50% chance that the coin will land heads.
- there is a 99% chance that my lottery ticket will lose.
- there is a very low chance that this table will spontaneously combust.

Credence, Beliefs, Chances

Lockean thesis: there is a threshold $0.5 \leq t \leq 1$ such that for all φ, $Bel(\varphi)$ if, and only if, $Cr(\varphi) > t$.

Chance-credence thesis: for all φ, if (you believe that) the chance of φ is c, then $Cr(\varphi) = c$.

Blue Bus Case

Suppose it is late at night...and an individual's car is hit by a bus. This individual cannot identify the bus, but she can establish that it is a blue bus, and she can prove as well that 80 percent of the blue buses in the city are operated by the Blue Bus Company, that 20 percent are operated by the Red Bus Company, and that there are no buses in the vicinity except those operated by one of these two companies. Moreover, each of the other elements of the case — negligence, causation, and, especially, the fact and the extent of the injury — is either stipulated or established to a virtual certainty.

F. Schauer. *Profiles, Probabilities, and Stereotyping*. Belknap Press, 2003.

Green Bus Case

Suppose it is late at night, and an individual's car is hit by a green bus. The two bus companies in the area, the Green Bus Company and the Yellow Bus Company, each operate 50 percent of the green busses. There is an eyewitness, who identifies the bus as belonging to the Green Bus Company (the two bus companies operate busses with distinctive shapes). It is night-time, and so her vision is not ideal: let us say she makes mistakes 25% of the time. All of the other elements of the case remain the same.

Let BB mean that a bus belonging to the Blue Bus Company hit the woman in the first case.

Let GB mean that a bus belonging to the Green Bus Company hit the woman in the second case.

Then, $Cr(BB) = 0.8$ and $Cr(GB) = 0.75$.

Only in the second case — the one with the lower credence — could the court judge that the plaintiff has won the suit.

This seems to suggest that the threshold view about the relationship between court verdicts and rational credence are false. What does this say about the Lockean thesis?

Lottery Paradox

H. Kyburg. *Probability and the Logic of Rational Belief*. Wesleyan University Press, 1961.

Lottery Paradox

Consider a fair lottery with 1,000 tickets and one prize.

The probability that a given ticket will win is 0.001 ($1/1,000$) and the probability that it will not win is 0.999.

"Surely if a sheer probability is ever sufficient to warrant the acceptance of a hypothesis, this is a case"

Let W_i mean that ticket i will win.

Assumption 1: For each ticket i, it is rational to have:

$$Cr(W_1) = Cr(W_2) = \cdots = Cr(W_{1000}) = \frac{1}{1000} = 0.001$$

and so,

$$Cr(\neg W_1) = Cr(\neg W_2) = \cdots = Cr(\neg W_{1000}) = \frac{999}{1000} = 0.999$$

Assumption 2: The Lockean thesis with threshold $t = 0.8$: For all φ, $Bel(\varphi)$ if and only if $Cr(\varphi) > 0.8$.

- Since $Cr(\neg W_1) > 0.8, Cr(\neg W_2) > 0.8, \cdots, Cr(\neg W_{1000}) > 0.8$ So, it is rational to believe each of the following:

$$\neg W_1, \neg W_2, \ldots, \neg W_{1000}$$

- Since you are *certain* that at least one ticket will win, $Cr(W_1 \vee W_2 \vee \cdots \vee W_{1000}) = 1$. So, it is rational to believe

$$W_1 \vee W_2 \vee \cdots \vee W_{1000}$$

.

Assumption 3: If it is rational to believe φ and rational to believe ψ, then it is rational to believe $\varphi \wedge \psi$.

Since it is rational to believe each of the following:

$$\neg W_1, \neg W_2, \ldots, \neg W_{1000}$$

it is rational to believe:

$$\neg W_1 \wedge \neg W_2 \wedge \cdots \wedge \neg W_{1000}$$

.

Assumption 4: It is not rational to believe a contradiction.

It is rational to believe:

$$\neg W_1 \wedge \neg W_2 \wedge \cdots \wedge \neg W_{1000}$$

It is rational to believe:

$$W_1 \vee W_2 \vee \cdots \vee W_{1000}$$

Assumption 4: It is not rational to believe a contradiction.

It is rational to believe:

$$\neg W_1 \wedge \neg W_2 \wedge \cdots \wedge \neg W_{1000}$$

It is rational to believe:

$$W_1 \vee W_2 \vee \cdots \vee W_{1000}$$

So, it is rational to believe:

$$(\neg W_1 \wedge \neg W_2 \wedge \cdots \wedge \neg W_{1000}) \wedge (W_1 \vee W_2 \vee \cdots \vee W_{1000})$$

But this formula is a contradiction!

The Lottery Paradox

- Which of the 4 assumptions should be given up?
  1. assigning a high credence that each ticket will lose
  2. the Lockean thesis
  3. if it is rational to believe φ and rational to believe ψ, then it is rational to believe $\varphi \wedge \psi$.
  4. it is not rational to believe a contradiction

- For any threshold, we can construct a lottery that produces the paradoxical result.