Reasoning for Humans: Clear Thinking in an Uncertain World

PHIL 171

Eric Pacuit

Department of Philosophy
University of Maryland
pacuit.org

Interpretations of probability

1. A measure of objective *evidential support* relations:
   E.g., "in light of the relevant seismological and geological data, California will *probably* experience a major earthquake this decade"

2. Someone's degree of confidence, or *graded belief*.
   E.g., "I am not sure that it will rain in College Park this week, but it *probably* will."

A. Hájek. *Interpretations of Probability*. The Stanford Encyclopedia of Philosophy (Fall 2019 Edition).

Actual (finite) frequency interpretation

Let P be an actual (non-empty, finite) population, let χ denote the set of (all) objects that actually have property χ.

Let $\#(S)$ be the number of objects in a set S. Using $\#(\cdot)$, we can define the actual frequency of χ in such a population P in the following way:

$$f_P(\chi) = \frac{\#(\chi \cap P)}{\#(P)}$$

Let X be the proposition that an (arbitrary) object $a \in P$ has property χ. Using f_P, we can define $Pr(X)$ as $f_P(\chi)$

Actual (finite) frequency interpretation: problems

- Frequencies are population-relative. If an object a is a member of multiple populations $P_1, P_2, \ldots, P_n$, then this may yield different values for $Pr_{P_1}(X), Pr_{P_2}(X), \ldots, Pr_{P_n}(X)$.

- Another peculiarity of finite actual frequencies is that they sometimes seem to be misleading about intuitive objective probabilities.

Hypothetical frequency interpretation

Probabilities are frequencies we would observe in a population — if that population were extended indefinitely (e.g., if we were to toss the coin infinitely many times)

- The *law of large numbers* ensures that (given certain underlying assumptions about the coin) the "settling down" we observe in many actual frequency cases (coin-tossing) will converge in the limit ($n \to \infty$).

Subjectivist interpretations

Our belief in propositions are often a matter of *degree*. That is, we often make judgments regarding the relative likelihood of events.

- I will eat pizza for dinner on Friday.
- Humans will travel to Mars.

I believe both propositions, but I think the first is *more likely* than the second

Conceptions of Belief

Binary: "all-out" or "full" belief. For any proposition φ, either you believe φ, disbelieve φ (i.e., believe $\neg\varphi$), or suspend judgement about φ (neither believe φ nor believe $\neg\varphi$).

Graded: "credences", beliefs come in degrees. We are *more confident* in some of our beliefs than in others.

Eric Schwitzgebel. *Belief*. In The Stanford Encyclopedia of Philosophy.

Franz Huber. *Formal Theories of Belief*. In The Stanford Encyclopedia of Philosophy.

Credence

A **credence function** assigns a number between 0 and 1 to each formula. For a formula φ, let $Cr(\varphi)$ be the credence for φ.

$Cr(\varphi)$ is a measure of an agent's belief that φ is true.

As we have seen, the credences of humans often deviate from the probability calculus (e.g., the conjunction fallacy). However, there are arguments that an agent's credence function *should* satisfy the laws of probability.

Constraints on Rational Beliefs/Credences

Let $Bel(\varphi)$ mean that φ is believed

What principles should *rational* belief satisfy?

- Consistency: If $Bel(\varphi)$, then φ is satisfiable.
- Aggregation: If $Bel(\varphi)$ and $Bel(\psi)$, then $Bel(\varphi \wedge \psi)$

Let $Cr(\varphi)$ be the credence that φ is true.

Rational credences (graded beliefs) should satisfy the axioms of probability

What is the relationship between full beliefs and credences?

D. Christensen. *Putting Logic in its Place*. Oxford University Press.

Bridge Principles

Probability 1: For all φ, $Bel(\varphi)$ if, and only if, $Cr(\varphi) = 1$

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The Lockean Thesis: There is some threshold $0.5 \leq t \leq 1$ such that for all φ, $Bel(\varphi)$ if, and only if, $Cr(\varphi) > t$