Reasoning for Humans: Clear Thinking in an Uncertain World

PHIL 171

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Conceptions of Belief

Binary: "all-out" or "full" belief. For any proposition X, either you believe X, disbelieve X (i.e., believe $\neg X$), or suspend judgement about X (neither believe X nor believe $\neg X$).

Graded: "credences", beliefs come in degrees. We are *more confident* in some of our beliefs than in others.

Eric Schwitzgebel. *Belief*. In The Stanford Encyclopedia of Philosophy.

Franz Huber. *Formal Theories of Belief*. In The Stanford Encyclopedia of Philosophy.

Constraints on Rational Beliefs/Credences

Let $Bel(X)$ mean that X is believed. Rational beliefs should satisfy the following principles:

- Consistency: If $Bel(X)$, then φ is satisfiable.
- Aggregation: If $Bel(X)$ and $Bel(Y)$, then $Bel(X \wedge Y)$

Constraints on Rational Beliefs/Credences

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Let $Cr(X)$ be the credence that X is true. Rational credences (graded beliefs) should satisfy the axioms of probability

- For any formula X, $Cr(\neg X) = 1 - Cr(X)$
- If X is a tautology, then $Cr(X) = 1$
- If X and Y are mutually exclusive, then $Cr(X \vee Y) = Cr(X) + Cr(Y)$

What is the relationship between full beliefs and credences?

D. Christensen. *Putting Logic in its Place*. Oxford University Press.

Bridge Principles

Probability 1: For all X, $Bel(X)$ if, and only if, $Cr(X) = 1$

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The Lockean Thesis: There is some threshold $0.5 \leq t \leq 1$ such that for all X, $Bel(X)$ if, and only if, $Cr(X) > t$

Credence, Beliefs, Chances

$Cr(X)$ the credence that X is true.

$Bel(X)$ the belief that X is true.

Chances: Consider beliefs/credences in propositions of the form "there is a chance c that X is true". For example,

- there is a 50% chance that the coin will land heads.
- there is a 99% chance that my lottery ticket will lose.
- there is a very low chance that this table will spontaneously combust.

Credence, Beliefs, Chances

Lockean thesis: there is a threshold $0.5 \leq t \leq 1$ such that for all φ, $Bel(X)$ if, and only if, $Cr(X) > t$.

Chance-credence thesis: for all X, if (you believe that) the chance of X is c, then $Cr(X) = c$.

Lottery Paradox

H. Kyburg. *Probability and the Logic of Rational Belief*. Wesleyan University Press, 1961.

Consider a fair lottery with 1,000 tickets and one prize.

The probability that a given ticket will win is 0.001 ($1/1,000$) and the probability that it will not win is 0.999.

"Surely if a sheer probability is ever sufficient to warrant the acceptance of a hypothesis, this is a case"

Let W_i mean that ticket i will win.

Assumption 1: For each ticket i, it is rational to have:

$$Cr(W_1) = Cr(W_2) = \cdots = Cr(W_{1000}) = \frac{1}{1000} = 0.001$$

and so,

$$Cr(\neg W_1) = Cr(\neg W_2) = \cdots = Cr(\neg W_{1000}) = \frac{999}{1000} = 0.999$$

Assumption 2: The Lockean thesis with threshold $t = 0.8$:
For all X, $Bel(X)$ if and only if $Cr(X) > 0.8$.

- Since $Cr(\neg W_1) > 0.8, Cr(\neg W_2) > 0.8, \cdots, Cr(\neg W_{1000}) > 0.8$. So, it is rational to believe each of the following:

$$\neg W_1, \neg W_2, \ldots, \neg W_{1000}$$

- Since you are *certain* that at least one ticket will win, $Cr(W_1 \vee W_2 \vee \cdots \vee W_{1000}) = 1$. So, it is rational to believe

$$W_1 \vee W_2 \vee \cdots \vee W_{1000}$$

.

Assumption 3: If it is rational to believe X and rational to believe Y, then it is rational to believe $X \wedge Y$.

Since it is rational to believe each of the following:

$$\neg W_1, \neg W_2, \ldots, \neg W_{1000}$$

it is rational to believe:

$$\neg W_1 \wedge \neg W_2 \wedge \cdots \wedge \neg W_{1000}$$

.

Assumption 4: It is not rational to believe a contradiction.

It is rational to believe:

$$\neg W_1 \wedge \neg W_2 \wedge \cdots \wedge \neg W_{1000}$$

It is rational to believe:

$$W_1 \vee W_2 \vee \cdots \vee W_{1000}$$

Assumption 4: It is not rational to believe a contradiction.

It is rational to believe:

$$\neg W_1 \wedge \neg W_2 \wedge \cdots \wedge \neg W_{1000}$$

It is rational to believe:

$$W_1 \vee W_2 \vee \cdots \vee W_{1000}$$

So, it is rational to believe:

$$(\neg W_1 \wedge \neg W_2 \wedge \cdots \wedge \neg W_{1000}) \wedge (W_1 \vee W_2 \vee \cdots \vee W_{1000})$$

But this formula is a contradiction!

The Lottery Paradox

- Which of the 4 assumptions should be given up?
  1. assigning a high credence that each ticket will lose
  2. the Lockean thesis
  3. if it is rational to believe X and rational to believe Y, then it is rational to believe $X \wedge Y$.
  4. it is not rational to believe a contradiction

- For any threshold, we can construct a lottery that produces the paradoxical result.