Reasoning for Humans: Clear Thinking in an Uncertain World

PHIL 171

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Stochastic Truth Table

	P	Q	$\cdots$
p_1	T	T	
p_2	T	F	
p_3	F	T	
p_4	F	F	

A **Stochastic Truth Table** is a truth table where there is a number assigned to each row such that:

(1) each number is greater than or equal to 0; and

(2) and the sum of all the numbers assigned to the rows is 1

Given a stochastic truth table, how do you determine the probability of a formula X?

$Pr(X) =$ the sum of the probabilities assigned to rows that make X true.

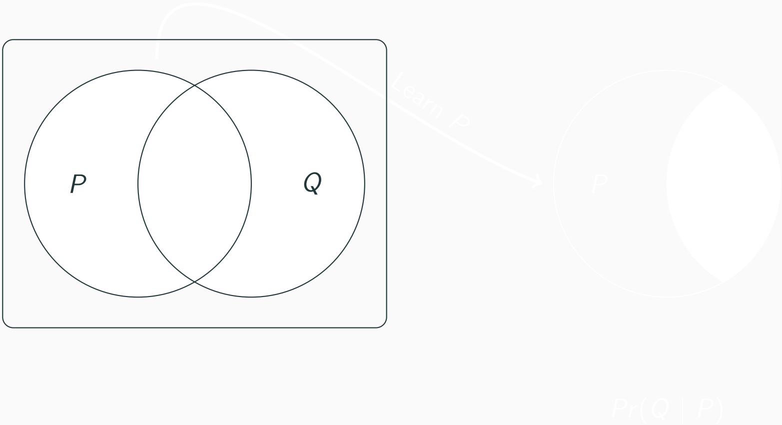
Conditional Probability

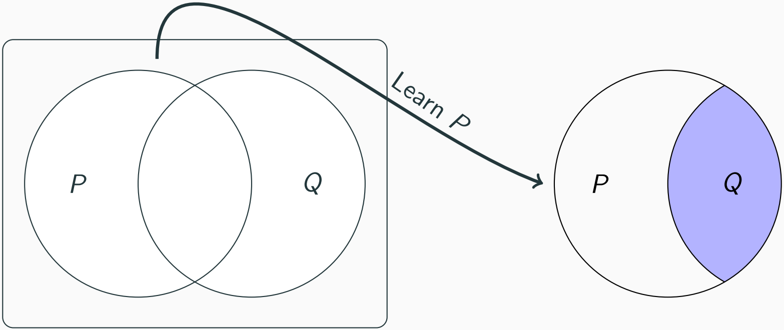
$Pr(X \mid Y)$: The probability of X **given that** Y.

Conditional probability is the concept of the probability of something *given* or *in the light of* some evidence or new information.

Example:

- the probability that the die lands 1, given that it lands odd, is 1/3
- the probability that it will rain tomorrow, given that there are dark clouds in the sky tomorrow morning, is high


P
Q
Learn P
P
Pr(Q | P)



$Pr(Q \mid P)$

Ratio Formula

$$Pr(X \mid Y) = \frac{Pr(X \wedge Y)}{Pr(Y)}$$

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$$Pr(X \mid Y) = \frac{Pr(X \wedge Y)}{Pr(Y)}$$

(If $Pr(Y) = 0$, then $Pr(X \mid Y)$ is undefined)

Kolmogrov Axioms

Given any stochastic truth table, for any formulas X and Y:

- $Pr(X) \geq 0$
- If X is a tautology, then $Pr(X) = 1$
- If X and Y are mutually exclusive, then $Pr(X \vee Y) = Pr(X) + Pr(Y)$

Laws of Probability

In any stochastic truth table, for all X, $Pr(\neg X) = 1 - Pr(X)$

In any stochastic truth table, for all X, if X is a contradiction, then $Pr(X) = 0$

In any stochastic truth table, for all X and Y, if $X \leftrightarrow Y$ is a tautology (i.e., X and Y are tautologically equivalent), then $Pr(X) = Pr(Y)$

Laws of Probability

In any stochastic truth table, for all X and Y,
$Pr(X) = Pr(X \wedge Y) + Pr(X \wedge \neg Y)$

In any stochastic truth table, for all X and Y,
$Pr(X \vee Y) = Pr(X) + Pr(Y) - Pr(X \wedge Y)$

Laws of Total Probability

In any stochastic truth table, for all X and Y,

$$Pr(X) = Pr(Y)Pr(X \mid Y) + Pr(\neg Y)Pr(X \mid \neg Y)$$

In any stochastic truth table, for all X and Y_1, Y_2 and Y_3, if

1. $Pr(Y_1 \vee Y_2 \vee Y_3) = 1$,
2. Y_1 and Y_2 are mutually exclusive, Y_2 and Y_3 are mutually exclusive, and Y_1 and Y_3 are mutually exclusive,

then

$$Pr(X) = Pr(Y_1)Pr(X \mid Y_1) + Pr(Y_2)Pr(X \mid Y_2) + Pr(Y_3)Pr(X \mid Y_3)$$

Conjunctions and Conditional Probability

In any stochastic truth table, for all X and Y,
$Pr(X \wedge Y) = Pr(X)Pr(Y \mid X)$

In any stochastic truth table, for all X, Y, and Z
$Pr(X \wedge Y \mid Z) = Pr(X \mid Z)Pr(Y \mid X \wedge Z)$

Conditionals in Stochastic Truth Tables

In any stochastic truth table, for all X and Y,
if $X \to Y$ is a tautology, then $Pr(X) \leq Pr(Y)$

Independence

Two formulas X and Y are **independent** given a stochastic truth table provided that $Pr(Y) = Pr(Y \mid X)$.

Equivalent ways of defining when two formulas X and Y are **independent** given a stochastic truth table:

- $Pr(X \wedge Y) = Pr(X)Pr(Y)$.
- $Pr(X) = Pr(X \mid Y)$.

Positive/Negative Relevance

X is **positively relevant** to Y (given a stochastic truth table) when $Pr(Y \mid X) > Pr(Y)$

X is **negatively relevant** to Y (given a stochastic truth table) when $Pr(Y \mid X) < Pr(Y)$

Evidential Support

X **evidentially supports** Y (given a stochastic truth table) when $Pr(Y \mid X) > 0.5$

Inductively Strong

The argument $P \Rightarrow C$ is inductively strong when:

1. P **evidentially supports** C: $Pr(C \mid P)$ is "high" (i.e., $Pr(C \mid P) > \frac{1}{2}$).
2. P is **positively relevant** to C: $Pr(C \mid P) > Pr(C)$
3. The argument is not deductively valid.

Posterior

Prior of H

$$Pr(H|E) = Pr(E|H)\frac{Pr(H)}{Pr(E)}$$

Likelihood

Prior of observing E

$$Pr(H \mid E) = Pr(E \mid H)\frac{Pr(H)}{Pr(E)}$$

$$Pr(H \mid E) = Pr(E \mid H)\frac{Pr(H)}{Pr(H)Pr(E \mid H) + Pr(\neg H)Pr(E \mid \neg H)}$$