# Reasoning for Humans: Clear Thinking in an Uncertain World

PHIL 171

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Posterior

Prior of $H$

$$Pr(H|E) = Pr(E|H)\frac{Pr(H)}{Pr(E)}$$

Likelihood

Prior of observing $E$

$$Pr(H \mid E) = Pr(E \mid H)\frac{Pr(H)}{Pr(E)}$$

$$Pr(H \mid E) = Pr(E \mid H)\frac{Pr(H)}{Pr(H)Pr(E \mid H) + Pr(\neg H)Pr(E \mid \neg H)}$$

# Bayes Theorem

Posterior

Prior of $H$

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Likelihood

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Suppose you select one card from a standard deck of cards.

$Q$ is "the card is a queen card" and $F$ is "the card is a face card".

What is $Pr(Q \mid F)$?

| | $Q$ | $F$ | $Q \wedge F$ |
|---|---|---|---|
| $\frac{4}{52}$ | T | T | T |
| $0$ | T | F | F |
| $\frac{8}{52}$ | F | T | F |
| $\frac{40}{52}$ | F | F | F |

$$Pr(Q \mid F) = \frac{Pr(Q \wedge F)}{Pr(F)} = \frac{\frac{4}{52}}{\frac{12}{52}} = \frac{4}{12} = \frac{1}{3}$$

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Note that:

$Pr(F \mid Q) = 1$

$Pr(F) = \frac{12}{52}$

$Pr(Q) = \frac{4}{52}$

$$Pr(Q \mid F) = Pr(F \mid Q)\frac{Pr(Q)}{Pr(F)} = 1 \times \frac{\frac{4}{52}}{\frac{12}{52}} = \frac{4}{12} = \frac{1}{3}$$

# Three Prisoner's Problem

Three prisoners $A, B$ and $C$ have been tried for murder and their verdicts will told to them tomorrow morning.

They know only that one of them will be declared guilty and will be executed while the others will be set free. The identity of the condemned prisoner is revealed to the very reliable prison guard, but not to the prisoners themselves.

Prisoner $A$ asks the guard "Please give this letter to one of my friends — to the one who is to be released. We both know that at least one of them will be released".

# Three Prisoner's Problem

An hour later, $A$ asks the guard "Can you tell me which of my friends you gave the letter to? It should give me no clue regarding my own status because, regardless of my fate, each of my friends had an equal chance of receiving my letter."

The guard told him that $B$ received his letter.

Prisoner $A$ then concluded that the probability that he will be released is $1/2$ (since the only ones without a verdict are $A$ and $C$).

# Three Prisoner's Problem

But, $A$ thinks to himself:

> *Before I talked to the guard my chance of being executed was 1 in 3. Now that he told me B has been released, only C and I remain, so my chances of being executed have gone from 33.33% to 50%. What happened? I made certain not to ask for any information relevant to my own fate...*

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Explain what is wrong with $A$'s reasoning.

# $A$'s reasoning

Consider the following events:

$G_A$: "Prisoner $A$ will be declared guilty" (we have $Pr(G_A) = 1/3$)

$I_B$: "Prisoner $B$ will be declared innocent" (we have $Pr(I_B) = 2/3$)

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Bayes Theorem:

$$Pr(G_A \mid I_B) =$$

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Bayes Theorem:

$$Pr(G_A \mid I_B) = Pr(I_B \mid G_A)\frac{Pr(G_A)}{Pr(I_B)} =$$

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Bayes Theorem:

$$Pr(G_A \mid I_B) = Pr(I_B \mid G_A)\frac{Pr(G_A)}{Pr(I_B)} = 1 \cdot \frac{1/3}{2/3} = 1/2$$