Reasoning for Humans: Clear Thinking in an Uncertain World

PHIL 171

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Valid Arguments

$$\varphi \to \psi, \chi \to \psi \models (\varphi \vee \chi) \to \psi$$

$$\varphi \to \psi, \neg\varphi \to \psi \models \psi$$

If $\varphi \models \psi$ and $\neg\varphi \models \psi$, then $\models \psi$ (that is, ψ is a tautology)

- P evidentially supports Q given R:
  $Pr(Q \mid P \wedge R) > 0.5$
- $P \wedge R$ evidentially supports Q:
  $Pr(Q \mid P \wedge R) > 0.5$

- P is positively relevant to Q given R:
  $Pr(Q \mid P \wedge R) > Pr(Q \mid R)$
- $P \wedge R$ is positively relevant to Q:
  $Pr(Q \mid P \wedge R) > Pr(Q)$

Sure-Thing Principle: If P is "good evidence" for Q given R and
P is "good evidence" for Q given $\neg R$,
then P is "good evidence" for Q

Unconditional Sure-Thing Principle: If $P \wedge R$ is "good evidence" for Q
and $P \wedge \neg R$ is "good evidence" for Q,
then P is "good evidence" for Q

Sure-Thing Principle

If P is "good evidence" for Q given R and P is "good evidence" for Q given $\neg R$, then P is "good evidence" for Q

1. If P evidentially supports Q given R and P evidentially supports Q given $\neg R$, then P evidentially supports Q

   If $Pr(Q \mid P \wedge R) > 0.5$ and $Pr(Q \mid P \wedge \neg R) > 0.5$,
   then $Pr(Q \mid P) > 0.5$

2. If P is positively relevant to Q given R and P is positively relevant to Q given $\neg R$, then P is positively relevant to Q

   If $Pr(Q \mid P \wedge R) > Pr(Q \mid R)$ and $Pr(Q \mid P \wedge \neg R) > Pr(Q \mid \neg R)$,
   then $Pr(Q \mid P) > Pr(Q)$

Sure-Thing Principle: Evidential Support

In any stochastic truth table (with $Pr(P) > 0$), if

1. $Pr(Q \mid P \wedge R) > 0.5$; and
2. $Pr(Q \mid P \wedge \neg R) > 0.5$

then $Pr(Q \mid P) > 0.5$

Unconditional Sure-Thing Principle: Positive Support

In any stochastic truth table (with $Pr(P) > 0$), if

1. $Pr(Q \mid P \wedge R) > Pr(Q)$; and
2. $Pr(Q \mid P \wedge \neg R) > Pr(Q)$

then $Pr(Q \mid P) > Pr(Q)$

Sure-Thing Principle: Positive Support

There is a stochastic truth table (with $Pr(P) > 0$) such that

1. $Pr(Q \mid P \wedge R) > Pr(Q \mid R)$; and
2. $Pr(Q \mid P \wedge \neg R) > Pr(Q \mid \neg R)$

but $Pr(Q \mid P) \leq Pr(Q)$

	P	Q	R
0.2	T	T	T
0.125	T	T	F
0.05	T	F	T
0.075	T	F	F
0.4	F	T	T
0	F	T	F
0.11	F	F	T
0.04	F	F	F

$$Pr(Q \mid P \wedge R) = 0.8$$
$$Pr(Q \mid R) \approx 0.789$$

$$Pr(Q \mid P \wedge \neg R) = 0.625$$
$$Pr(Q \mid \neg R) \approx 0.521$$

$$Pr(Q \mid P) \approx 0.722$$
$$Pr(Q) = 0.725$$

Simpson's Paradox

Suppose that a University is hiring in Philosophy and Mathematics. 13 men and 13 women apply for jobs.

	Men	Women	
Mathematics	1/5	2/8	*success rate better for women*
Philosophy	6/8	4/5	*success rate better for women*

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Philosophy	6/8	4/5	*success rate better for women*
University	7/13	6/13	*success rate better for men*

How can it be that each Department favors women applicants, and yet overall men fare better than women?

G. Malinas and J. Bigelow. *Simpson's Paradox*. The Stanford Encyclopedia of Philosophy, Fall 2012 Edition, Edward N. Zalta (ed.).

Summary

- If $\varphi \models \psi$ and $\neg\varphi \models \psi$, then $\models \psi$.

- Both the Sure-Thing Principle and the Unconditional Sure-Thing Principle holds for evidential support: If $Pr(Q \mid P \wedge R) > 0.5$ and $Pr(Q \mid P \wedge \neg R) > 0.5$, then $Pr(Q \mid P) > 0.5$

- The Sure-Thing Principle does not hold for positive relevance:

  There are stochastic truth tables where: $Pr(Q \mid P \wedge R) > Pr(Q \mid R)$ and $Pr(Q \mid P \wedge \neg R) > Pr(Q \mid \neg R)$, but $Pr(Q \mid P) \leq Pr(Q)$

- The Unconditional Sure-Thing Principle does hold for positive relevance: If $Pr(Q \mid P \wedge R) > Pr(Q)$ and $Pr(Q \mid P \wedge \neg R) > Pr(Q)$, then $Pr(Q \mid P) > Pr(Q)$

Bayes Theorem

Conditional Probability

The probability of H *given* E, $Pr(H \mid E)$, is defined to be

$$Pr(H \mid E) = \frac{Pr(H \wedge E)}{Pr(E)}.$$

provided $Pr(E) > 0$.

Bayes Theorem

$$Pr(H \mid E) = Pr(E \mid H)\frac{Pr(H)}{Pr(E)}$$

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$$Pr(H \mid E) = Pr(E \mid H)\frac{Pr(H)}{Pr(E)}$$

Bayes theorem is important because it expresses the quantity $Pr(H \mid E)$ (the probability of a hypothesis H given the evidence E) —which is something people often find hard to assess—in terms of quantities that can be drawn directly from experiential knowledge.

Example: Suppose you are in a casino and you hear a person at the next gambling table announce "Twelve". We want to know wether he was rolling a pair of dice or a roulette wheel.

That is, compare $Pr(Dice \mid Twelve)$ with $Pr(Roulette \mid Twelve)$.

Based on our background knowledge of gambling we have $Pr(Twelve \mid Dice) = 1/36$ and $Pr(Twelve \mid Roulette) = 1/38$.

Based on our observations about the casino, we can judge the prior probabilities $Pr(Dice)$ and $Pr(Roulette)$.

But this is now enough to *calculate* the required probabilities.

Posterior

Prior of H

$$Pr(H|E) = Pr(E|H)\frac{Pr(H)}{Pr(E)}$$

Likelihood

Prior of observing E

$$Pr(H \mid E) = Pr(E \mid H)\frac{Pr(H)}{Pr(E)}$$

$$Pr(H \mid E) = Pr(E \mid H)\frac{Pr(H)}{Pr(H)Pr(E \mid H) + Pr(\neg H)Pr(E \mid \neg H)}$$

Non-Monotonicity

- If $\varphi \Rightarrow \psi$ is valid, then $\varphi, \chi \Rightarrow \psi$ is valid.
- If $\varphi \rightarrow \psi$ is a tautology, then $(\varphi \wedge \chi) \rightarrow \psi$ is a tautology.
- There are formulas φ, ψ, and χ, a stochastic truth table and $p \geq 0.5$ such that $Pr(\psi \mid \varphi) > p$ and $Pr(\psi \mid \varphi \wedge \chi) < p$.
- There are formulas φ, ψ, and χ, and a stochastic truth table such that $Pr(\psi \mid \varphi) > Pr(\psi)$ and $Pr(\psi \mid \varphi \wedge \chi) < Pr(\psi)$.

Conjunction Rule

- A (deductively) valid argument: $E \to (P \wedge Q) \models E \to P$
- If E evidentially supports $P \wedge Q$, then E evidentially supports P.

  In any stochastic truth table, if $Pr(P \wedge Q \mid E) > \frac{1}{2}$, then $Pr(P \mid E) > \frac{1}{2}$. In fact,

  $$Pr(P \mid E) \geq Pr(P \wedge Q \mid E)$$

- E may be positively relevant for $P \wedge Q$ without being positively relevant for P:

  There are stochastic truth tables where $Pr(P \wedge Q \mid E) > Pr(P \wedge Q)$ but $Pr(P \mid E) > Pr(P)$.

Sure-Thing Principle

- If $\varphi \models \psi$ and $\neg\varphi \models \psi$, then $\models \psi$.

- Both the Sure-Thing Principle and the Unconditional Sure-Thing Principle holds for evidential support: If $Pr(Q \mid P \wedge R) > 0.5$ and $Pr(Q \mid P \wedge \neg R) > 0.5$, then $Pr(Q \mid P) > 0.5$

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- The Unconditional Sure-Thing Principle does hold for positive relevance: If $Pr(Q \mid P \wedge R) > Pr(Q)$ and $Pr(Q \mid P \wedge \neg R) > Pr(Q)$, then $Pr(Q \mid P) > Pr(Q)$