Reasoning for Humans: Clear Thinking in an Uncertain World

PHIL 171

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Independence

Two formulas X and Y are **independent** given a stochastic truth table provided that $Pr(Y) = Pr(Y \mid X)$.

Equivalent ways of defining when two formulas X and Y are **independent** given a stochastic truth table:

- $Pr(X \wedge Y) = Pr(X)Pr(Y)$.
- $Pr(X) = Pr(X \mid Y)$.

Positive/Negative Relevance

X is **positively relevant** to Y (given a stochastic truth table) when $Pr(Y \mid X) > Pr(Y)$

X is **negatively relevant** to Y (given a stochastic truth table) when $Pr(Y \mid X) < Pr(Y)$

$P \Rightarrow C$ is inductively strong when:

1. P **evidentially supports** C: $Pr(C \mid P)$ is "high" (i.e., $Pr(C \mid P) > \frac{1}{2}$).
2. P is **positively relevant** to C: $Pr(C \mid P) > Pr(C)$
3. The argument is not deductively valid.

Valid arguments are **monotonic**:

- If $X \Rightarrow Y$ is valid, then $X, Z \Rightarrow Y$ is valid.
- If $X \rightarrow Y$ is a tautology, then $(X \wedge Z) \rightarrow Y$ is a tautology.

What about inductively strong arguments?

If $X \Rightarrow Y$ is inductively strong, then is $X, Z \Rightarrow Y$ inductively strong?

If $X \Rightarrow Y$ is inductively strong, then is $X, Z \Rightarrow Y$ inductively strong?

1. For any stochastic truth table, if $Pr(Y \mid X) > 0.5$, then $Pr(Y \mid X \wedge Z) > 0.5$
2. For any stochastic truth table, if $Pr(Y \mid X) > Pr(Y)$, then $Pr(Y \mid X \wedge Z) > Pr(Y)$

There are formulas X, Y, and Z and a stochastic truth table such that

$$Pr(Y \mid X) > 0.5 \qquad \text{and} \qquad Pr(Y \mid X \wedge Z) < 0.5$$

prob	P	Q	R
0.1	T	T	T
0.25	T	T	F
0.2	T	F	T
0.05	T	F	F
0.05	F	T	T
0.05	F	T	F
0.2	F	F	T
0.1	F	F	F

$$Pr(Q \mid P) = \frac{Pr(Q \wedge P)}{Pr(P)} = \frac{0.35}{0.6} \approx 0.58 > 0.5$$

$$Pr(Q \mid P \wedge R) = \frac{Pr(Q \wedge P \wedge R)}{Pr(P \wedge R)} = \frac{0.1}{0.3} \approx 0.33 < 0.5$$

Tweety is a bird. Therefore, Tweety flies.
$(Pr(Ft \mid Bt) > 0.5)$

Tweety is a bird. Tweety is a penguin. Therefore, Tweety flies.
$(Pr(Ft \mid Bt \wedge Pt) < 0.5)$

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0.2	F	F	T
0.1	F	F	F

$$Pr(Q \mid P) = \frac{Pr(Q \wedge P)}{Pr(P)} = \frac{0.35}{0.6} \approx 0.58 > 0.45 = Pr(Q)$$

$$Pr(Q \mid P \wedge R) = \frac{Pr(Q \wedge P \wedge R)}{Pr(P \wedge R)} = \frac{0.1}{0.3} \approx 0.33 < 0.45 = Pr(Q)$$

Summary

- If $X \Rightarrow Y$ is valid, then $X, Z \Rightarrow Y$ is valid.
- If $X \rightarrow Y$ is a tautology, then $(X \wedge Z) \rightarrow Y$ is a tautology.

- There are formulas X, Y, and Z, a stochastic truth table and $p \geq 0.5$ such that $Pr(Y \mid X) > p$ and $Pr(Y \mid X \wedge Z) < p$.

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Conjunction Fallacy

Linda is 31 years old, single, outspoken, and very bright. She majored in philosophy. As a student, she was deeply concerned with issues of discrimination and social justice, and also participated in anti-nuclear demonstrations.

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Typically, many people asked say that 2 is more probable than 1.

A. Tversky and D. Kahneman. *Extensions versus intuitive reasoning: The conjunction fallacy in probability judgment*. Psychological Review 90 (4): 293 - 315, 1983.

Conjunction Fallacy

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1. Linda is a bank teller. B
2. Linda is a bank teller and is active in the feminist movement. $B \wedge F$

$Pr(B \mid E) \geq Pr(B \wedge F \mid E)$

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But, E **is positively relevant for** $B \wedge F$ **(and less so than to** B**)**

Conjunction Principle

If E is "good evidence" for $P \wedge Q$, then it is "good evidence" for P.

$$\frac{E \rightarrow (P \wedge Q)}{\therefore \quad E \rightarrow P}$$

P	Q	E	$P \wedge Q$	$E \to (P \wedge Q)$	$E \to P$
T	T	T	T	T	T
T	T	F	T	T	T
T	F	T	F	F	T
T	F	F	F	T	T
F	T	T	F	F	F
F	T	F	F	T	T
F	F	T	F	F	F
F	F	F	F	T	T

$$(E \to (P \wedge Q)) \models (E \to P)$$

Evidential Support vs. Relevance

1. Is it true that if E evidentially supports $P \wedge Q$, then E evidentially supports P (If $Pr(P \wedge Q \mid E)$ is high, then so is $Pr(P \mid E)$)?

   Yes: If $Pr(P \wedge Q \mid E) > \frac{1}{2}$, then $Pr(P \mid E) > \frac{1}{2}$. In fact, for all X, Y, Z, $Pr(X \mid Z) \geq Pr(X \wedge Y \mid Z)$.

2. Is it true that if E is positively relevant for $P \wedge Q$, then E is positively relevant for P?

   No: E can be positively relevant for $P \wedge Q$ without being positively relevant to P. That is, $Pr(P \wedge Q \mid E) > Pr(P \wedge Q)$ does not necessarily imply that $Pr(P \mid E) > Pr(P)$.

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P	Q	E	$P \wedge Q$	$E \rightarrow (P \wedge Q)$	$E \rightarrow P$
T	T	T	T	T	T
T	T	F	T	T	T
T	F	T	F	F	T
T	F	F	F	T	T
F	T	T	F	F	F
F	T	F	F	T	T
F	F	T	F	F	F
F	F	F	F	T	T

$$E \rightarrow (P \wedge Q) \models E \rightarrow P$$

	P	Q	E	$P \wedge Q$	$E \rightarrow (P \wedge Q)$	$E \rightarrow P$
p_1	T	T	T	T	T	T
p_2	T	T	F	T	T	T
p_3	T	F	T	F	F	T
p_4	T	F	F	F	T	T
p_5	F	T	T	F	F	F
p_6	F	T	F	F	T	T
p_7	F	F	T	F	F	F
p_8	F	F	F	F	T	T

$$Pr(P \mid E) \geq Pr(P \wedge Q \mid E)$$

	P	Q	E	$P \wedge Q$	$E \rightarrow (P \wedge Q)$	$E \rightarrow P$
$\frac{p_1}{p_1+p_3+p_5+p_7}$	T	T	T	T	T	T
0	T	T	F	T	T	T
$\frac{p_3}{p_1+p_3+p_5+p_7}$	T	F	T	F	F	T
0	T	F	F	F	T	T
$\frac{p_5}{p_1+p_3+p_5+p_7}$	F	T	T	F	F	F
0	F	T	F	F	T	T
$\frac{p_7}{p_1+p_3+p_5+p_7}$	F	F	T	F	F	F
0	F	F	F	F	T	T

$$Pr(P \mid E) \geq Pr(P \wedge Q \mid E)$$

	P	Q	E	$P \wedge Q$	$E \to (P \wedge Q)$	$E \to P$
$\frac{p_1}{p_1+p_3+p_5+p_7}$	T	T	T	T	T	T
0	T	T	F	T	T	T
$\frac{p_3}{p_1+p_3+p_5+p_7}$	T	F	T	F	F	T
0	T	F	F	F	T	T
$\frac{p_5}{p_1+p_3+p_5+p_7}$	F	T	T	F	F	F
0	F	T	F	F	T	T
$\frac{p_7}{p_1+p_3+p_5+p_7}$	F	F	T	F	F	F
0	F	F	F	F	T	T

$$Pr(P \mid E) = \frac{p_1+p_3}{p_1+p_3+p_5+p_7} \geq \frac{p_1}{p_1+p_3+p_5+p_7} = Pr(P \wedge Q \mid E)$$

B: The card is black.

A: The card is an ace.

S: The card is a spade.

$Pr(A \wedge S \mid B) > Pr(A \wedge S)$, but $Pr(A \mid B) = Pr(A)$.

	A	S	B	$A \wedge S$	$B \rightarrow (A \wedge S)$	$B \rightarrow A$
$\frac{1}{52}$	T	T	T	T	T	T
0	T	T	F	T	T	T
$\frac{1}{52}$	T	F	T	F	F	T
$\frac{2}{52}$	T	F	F	F	T	T
$\frac{12}{52}$	F	T	T	F	F	F
0	F	T	F	F	T	T
$\frac{12}{52}$	F	F	T	F	F	F
$\frac{24}{52}$	F	F	F	F	T	T

$Pr(A \wedge S \mid B) = \frac{1}{26} > Pr(A \wedge S) = \frac{1}{52}$, but $Pr(A \mid B) = Pr(A) = \frac{2}{26}$

	A	S	B	$A \wedge S$	$B \rightarrow (A \wedge S)$	$B \rightarrow A$
$\frac{1}{52}$	T	T	T	T	T	T
0	T	T	F	T	T	T
$\frac{1}{52}$	T	F	T	F	F	T
$\frac{2}{52}$	T	F	F	F	T	T
$\frac{12}{52}$	F	T	T	F	F	F
0	F	T	F	F	T	T
$\frac{12}{52}$	F	F	T	F	F	F
$\frac{24}{52}$	F	F	F	F	T	T

$Pr(A \wedge S \mid B) = \frac{1}{26} > Pr(A \wedge S) = \frac{1}{52}$, but $Pr(A \mid B) = Pr(A) = \frac{1}{13}$

	A	S	B	$A \wedge S$	$B \rightarrow (A \wedge S)$	$B \rightarrow A$
$\frac{1}{26}$	T	T	T	T	T	T
0	T	T	F	T	T	T
$\frac{1}{26}$	T	F	T	F	F	T
0	T	F	F	F	T	T
$\frac{12}{26}$	F	T	T	F	F	F
0	F	T	F	F	T	T
$\frac{12}{26}$	F	F	T	F	F	F
0	F	F	F	F	T	T

$Pr(A \wedge S \mid B) = \frac{1}{26} > Pr(A \wedge S) = \frac{1}{52}$, but $Pr(A \mid B) = Pr(A) = \frac{1}{13}$

Summary

- A (deductively) valid argument: $E \to (P \wedge Q) \models E \to P$

- If E evidentially supports $P \wedge Q$, then E evidentially supports P.

  If $Pr(P \wedge Q \mid E) > \frac{1}{2}$, then $Pr(P \mid E) > \frac{1}{2}$. In fact,

$$Pr(P \mid E) \geq Pr(P \wedge Q \mid E)$$

- However, E may be positively relevant for $P \wedge Q$ without being positively relevant for P:

  $Pr(P \wedge Q \mid E) > Pr(P \wedge Q)$ does not necessarily imply that $Pr(P \mid E) > Pr(P)$.