Reasoning for Humans: Clear Thinking in an Uncertain World

PHIL 171

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$$\varphi \Rightarrow \psi$$

When is an argument inductively strong?

1. φ **evidentially supports** ψ: $Pr(\psi \mid \varphi)$ is "high" (i.e., $Pr(\psi \mid \varphi) > \frac{1}{2}$).
2. φ is **positively relevant** to ψ: $Pr(\psi \mid \varphi) > Pr(\psi)$
3. The argument is not deductively valid.

Correlation vs. Independence

φ and ψ are independent if $Pr(\psi \mid \varphi) = Pr(\psi)$. Equivalent ways of defining when two formulas φ and ψ are **independent** given a stochastic truth table:

- $Pr(\varphi \wedge \psi) = Pr(\varphi)Pr(\psi)$.
- $Pr(\psi) = Pr(\psi \mid \varphi)$.

φ is positively relevant to ψ if $Pr(\psi \mid \varphi) > Pr(\psi)$

φ is negatively relevant to ψ if $Pr(\psi \mid \varphi) < Pr(\psi)$

Valid arguments are **monotonic**:

- If $\varphi \Rightarrow \psi$ is valid, then $\varphi, \chi \Rightarrow \psi$ is valid.
- If $\varphi \rightarrow \psi$ is a tautology, then $(\varphi \wedge \chi) \rightarrow \psi$ is a tautology.

What about inductively strong arguments?

If $\varphi \Rightarrow \psi$ is inductively strong, then is $\varphi, \chi \Rightarrow \psi$ inductively strong?

If $\varphi \Rightarrow \psi$ is inductively strong, then is $\varphi, \chi \Rightarrow \psi$ inductively strong?

1. For any stochastic truth table, if $Pr(\psi \mid \varphi) > 0.5$, then $Pr(\psi \mid \varphi \wedge \chi) > 0.5$

2. For any stochastic truth table, if $Pr(\psi \mid \varphi) > Pr(\psi)$, then $Pr(\psi \mid \varphi \wedge \chi) > Pr(\psi)$

There are formulas φ, ψ, and χ and a stochastic truth table such that

$$Pr(\psi \mid \varphi) > 0.5 \qquad \text{and} \qquad Pr(\psi \mid \varphi \wedge \chi) < 0.5$$

prob	P	Q	R
0.1	T	T	T
0.25	T	T	F
0.2	T	F	T
0.05	T	F	F
0.05	F	T	T
0.05	F	T	F
0.2	F	F	T
0.1	F	F	F

$$Pr(Q \mid P) = \frac{Pr(Q \wedge P)}{Pr(P)} = \frac{0.35}{0.6} \approx 0.58 > 0.5$$

$$Pr(Q \mid P \wedge R) = \frac{Pr(Q \wedge P \wedge R)}{Pr(P \wedge R)} = \frac{0.1}{0.3} \approx 0.33 < 0.5$$

Tweety is a bird. Therefore, Tweety flies.
($Pr(Ft \mid Bt) > 0.5$)

Tweety is a bird. Tweety is a penguin. Therefore, Tweety flies.
($Pr(Ft \mid Bt \wedge Pt) < 0.5$)

There are formulas φ, ψ, and χ and a stochastic truth table such that

$$Pr(\psi \mid \varphi) > Pr(\psi) \qquad \text{and} \qquad Pr(\psi \mid \varphi \wedge \chi) < Pr(\psi)$$

prob	P	Q	R
0.1	T	T	T
0.25	T	T	F
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$$Pr(Q \mid P) = \frac{Pr(Q \wedge P)}{Pr(P)} = \frac{0.35}{0.6} \approx 0.58 > 0.45 = Pr(Q)$$

$$Pr(Q \mid P \wedge R) = \frac{Pr(Q \wedge P \wedge R)}{Pr(P \wedge R)} = \frac{0.1}{0.3} \approx 0.33 < 0.45 = Pr(Q)$$

Summary

- If $\varphi \Rightarrow \psi$ is valid, then $\varphi, \chi \Rightarrow \psi$ is valid.
- If $\varphi \rightarrow \psi$ is a tautology, then $(\varphi \land \chi) \rightarrow \psi$ is a tautology.
- There are formulas φ, ψ, and χ, a stochastic truth table and $p \geq 0.5$ such that $Pr(\psi \mid \varphi) > p$ and $Pr(\psi \mid \varphi \land \chi) < p$.
- There are formulas φ, ψ, and χ, and a stochastic truth table such that $Pr(\psi \mid \varphi) > Pr(\psi)$ and $Pr(\psi \mid \varphi \land \chi) < Pr(\psi)$.