Reasoning for Humans: Clear Thinking in an Uncertain World

PHIL 171

Eric Pacuit

Department of Philosophy
University of Maryland
pacuit.org

$$\frac{P}{\therefore C}$$

When is an argument inductively strong?

...when it is *improbable* that its conclusion is false while its premises are true.

- when $P \rightarrow C$ is probable? No!
- when $P \land \neg C$ is improbable? No!

The inductive probability of an argument is determined by the evidential relation between its premises and its conclusion, not by the likelihood of the truth of its premises alone or the likelihood of the truth of its conclusion alone.

Every day so far this semester, Ann brought her laptop to class.
Therefore, Ann will bring her laptop to the next class.

Every day so far this semester, Ann brought her laptop to class. Therefore, Ann will bring her laptop to the next class.

Ann said that she saw Bob steal the laptop. Therefore, Bob stole the laptop

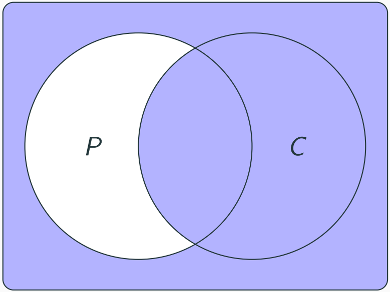
Ten eyewitnesses said that they saw Bob steal the laptop. Therefore, Bob stole the laptop.

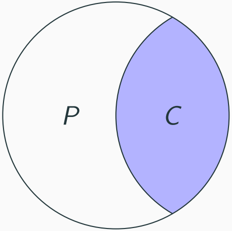
Conditional Probability

The inductive probability of an argument is the probability that its conclusion is true given that its premises are true.

$$Pr(C \mid P) = \frac{Pr(C \wedge P)}{Pr(P)}$$

$$\frac{P}{\therefore\ C}$$





$Pr(P \rightarrow C)$

$Pr(C \mid P)$

$$\frac{P}{\therefore C}$$

When is an argument inductively strong?

1. P **evidentially supports** C: $Pr(C \mid P)$ is “high” (i.e., $Pr(C \mid P) > \frac{1}{2}$).
2. The argument is not valid.
3. Are we missing any other conditions?

Bob (who is male) is taking a birth control pill.

$\therefore$ Bob (who is male) is not going to get pregnant.

$$Pr(C \mid P) = 1 = Pr(C)$$

$$\frac{\text{Bob (who is male) is taking a birth control pill.}}{\therefore \text{ Bob (who is male) is not going to get pregnant.}}$$

$$Pr(C \mid P) = 1 = Pr(C)$$

P is *irrelevant* to the truth of C

Independence

Two formulas X and Y are **independent** given a stochastic truth table provided that $Pr(Y) = Pr(Y \mid X)$.

Equivalent ways of defining when two formulas X and Y are **independent** given a stochastic truth table:

- $Pr(X \wedge Y) = Pr(X)Pr(Y)$.
- $Pr(X) = Pr(X \mid Y)$.

Example

Suppose that you are flipping two coins. Let P be true when the first coin landed heads and Q be true when the second coin landed heads.

Example

Suppose you flip the coins many times and record the percentage of the four possible coin flips resulting in the following stochastic truth table:

	P	Q	$P \wedge Q$
0.25	T	T	T
0.25	T	F	F
0.25	F	T	F
0.25	F	F	F

P and Q are independent:

- $Pr(P) = 0.5 = \frac{0.25}{0.5} = \frac{Pr(P \wedge Q)}{Pr(Q)} = Pr(P \mid Q)$
- $Pr(P \wedge Q) = 0.25 = 0.5 \times 0.5 = Pr(P) \times Pr(Q)$
- $Pr(Q) = 0.5 = \frac{0.25}{0.5} = \frac{Pr(Q \wedge P)}{Pr(P)} = Pr(Q \mid P)$

Example

Suppose you flip the coins many times and record the percentage of the four possible coin flips resulting in the following stochastic truth table:

	P	Q	$P \wedge Q$
0.4	T	T	T
0.1	T	F	F
0.1	F	T	F
0.4	F	F	F

P and Q are **not** independent:

- $Pr(P) = 0.5 \neq 0.8 = \frac{0.4}{0.5} = \frac{Pr(P \wedge Q)}{Pr(Q)} = Pr(P \mid Q)$
- $Pr(P \wedge Q) = 0.4 \neq 0.25 = 0.5 \times 0.5 = Pr(P) \times Pr(Q)$
- $Pr(Q) = 0.5 \neq 0.8 = \frac{0.4}{0.5} = \frac{Pr(Q \wedge P)}{Pr(P)} = Pr(Q \mid P)$

Positive/Negative Relevance

X is **positively relevant** to Y (given a stochastic truth table) when $Pr(Y \mid X) > Pr(Y)$

X is **negatively relevant** to Y (given a stochastic truth table) when $Pr(Y \mid X) < Pr(Y)$

$$\frac{P}{\therefore C}$$

When is an argument inductively strong?

1. P **evidentially supports** C: $Pr(C \mid P)$ is “high” (i.e., $Pr(C \mid P) > \frac{1}{2}$).
2. P is **positively relevant** to C: $Pr(C \mid P) > Pr(C)$
3. The argument is not deductively valid.