Reasoning for Humans: Clear Thinking in an Uncertain World

PHIL 171

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$$\begin{array}{c} P_1 \\ P_2 \\ P_3 \\ \hline \therefore C \end{array}$$

When is an argument inductively strong?

1. $P_1 \wedge P_2 \wedge P_3$ **evidentially supports** C: $Pr(C \mid (P_1 \wedge P_2 \wedge P_3))$ is "high" (i.e., $Pr(C \mid (P_1 \wedge P_2 \wedge P_3)) > \frac{1}{2}$).
2. $P_1 \wedge P_2 \wedge P_3$ is **positively relevant** to C: $Pr(C \mid (P_1 \wedge P_2 \wedge P_3)) > Pr(C)$
3. The argument is not deductively valid.

Evidential Support

- Whether or not φ evidentially supports ψ depends on the stochastic truth table.
- In any stochastic truth table, if φ evidentially supports $\psi \wedge \chi$, then φ evidentially supports ψ
- In any stochastic truth table, if φ evidentially supports ψ, then φ evidentially supports $\psi \vee \chi$
- If ψ is a tautology, then for any formula φ, φ evidentially supports ψ

Correlation vs. Independence

φ and ψ are independent if $Pr(\psi \mid \varphi) = Pr(\psi)$

φ is positively relevant to ψ if $Pr(\psi \mid \varphi) > Pr(\psi)$

φ is negatively relevant to ψ if $Pr(\psi \mid \varphi) < Pr(\psi)$

Conjunctions and Conditional Probability

In any stochastic truth table, for all φ and ψ,
$Pr(\varphi \wedge \psi) = Pr(\varphi)Pr(\psi \mid \varphi)$

In any stochastic truth table, for all φ, ψ, and χ
$Pr(\varphi \wedge \psi \mid \chi) = Pr(\varphi \mid \chi)Pr(\psi \mid \varphi \wedge \chi)$

Independence

Two formulas φ and ψ are **independent** given a stochastic truth table provided that $Pr(\varphi) = Pr(\varphi \mid \psi)$.

Equivalent ways of defining when two formulas φ and ψ are **independent** given a stochastic truth table:

- $Pr(\varphi \wedge \psi) = Pr(\varphi)Pr(\psi)$.
- $Pr(\psi) = Pr(\psi \mid \varphi)$.

Example

Suppose that you are flipping two coins. Let P be true when the first coin landed heads and Q be true when the second coin landed heads.

Example

Suppose you flip the coins many times and record the percentage of the four possible coin flips resulting in the following stochastic truth table:

	P	Q	$P \wedge Q$
0.25	T	T	T
0.25	T	F	F
0.25	F	T	F
0.25	F	F	F

P and Q are independent:

- $Pr(P) = 0.5 = \frac{0.25}{0.5} = \frac{Pr(P \wedge Q)}{Pr(Q)} = Pr(P \mid Q)$
- $Pr(P \wedge Q) = 0.25 = 0.5 \times 0.5 = Pr(P) \times Pr(Q)$
- $Pr(Q) = 0.5 = \frac{0.25}{0.5} = \frac{Pr(Q \wedge P)}{Pr(P)} = Pr(Q \mid P)$

Example

Suppose you flip the coins many times and record the percentage of the four possible coin flips resulting in the following stochastic truth table:

	P	Q	$P \wedge Q$
0.4	T	T	T
0.1	T	F	F
0.1	F	T	F
0.4	F	F	F

P and Q are **not** independent:

- $Pr(P) = 0.5 \neq 0.8 = \frac{0.4}{0.5} = \frac{Pr(P \wedge Q)}{Pr(Q)} = Pr(P \mid Q)$
- $Pr(P \wedge Q) = 0.4 \neq 0.25 = 0.5 \times 0.5 = Pr(P) \times Pr(Q)$
- $Pr(Q) = 0.5 \neq 0.8 = \frac{0.4}{0.5} = \frac{Pr(Q \wedge P)}{Pr(P)} = Pr(Q \mid P)$

Inductive Strength

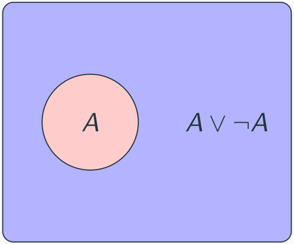
Argument 1	Argument 2
$A \vee B$	$A \vee \neg A$
$\therefore \quad A$	$\therefore \quad A$

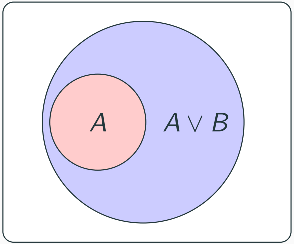
- Argument 1: You will get either an A or B in PHIL 171. Therefore, you will get an A in PHIL 171.
- Argument 2: Either you will get an A or you will not get an A in PHIL 171. Therefore, you will get an A in PHIL 171.
- Argument 1 is not valid, and Argument 2 is not valid.
- Argument 1 is *stronger* than Argument 2.

Inductive Strength

$$\frac{A \vee \neg A}{\therefore\ A}$$

$$\frac{A \vee B}{\therefore\ A}$$





Valid arguments are **monotonic**:

- If $\varphi \Rightarrow \psi$ is valid, then $\varphi, \chi \Rightarrow \psi$ is valid.
- If $\varphi \rightarrow \psi$ is a tautology, then $(\varphi \wedge \chi) \rightarrow \psi$ is a tautology.

What about inductively strong arguments?

If $\varphi \Rightarrow \psi$ is inductively strong, then is $\varphi, \chi \Rightarrow \psi$ inductively strong?

If $\varphi \Rightarrow \psi$ is inductively strong, then is $\varphi, \chi \Rightarrow \psi$ inductively strong?

1. For any stochastic truth table, if $Pr(\psi \mid \varphi) > 0.5$, then $Pr(\psi \mid \varphi \wedge \chi) > 0.5$
2. For any stochastic truth table, if $Pr(\psi \mid \varphi) > Pr(\psi)$, then $Pr(\psi \mid \varphi \wedge \chi) > Pr(\psi)$

There are formulas φ, ψ, and χ and a stochastic truth table such that

$$Pr(\psi \mid \varphi) > 0.5 \qquad \text{and} \qquad Pr(\psi \mid \varphi \wedge \chi) < 0.5$$

prob	P	Q	R
0.1	T	T	T
0.25	T	T	F
0.2	T	F	T
0.05	T	F	F
0.05	F	T	T
0.05	F	T	F
0.2	F	F	T
0.1	F	F	F

$$Pr(Q \mid P) = \frac{Pr(Q \wedge P)}{Pr(P)} = \frac{0.35}{0.6} \approx 0.58 > 0.5$$

$$Pr(Q \mid P \wedge R) = \frac{Pr(Q \wedge P \wedge R)}{Pr(P \wedge R)} = \frac{0.1}{0.3} \approx 0.33 < 0.5$$

Tweety is a bird. Therefore, Tweety flies.
$(Pr(Ft \mid Bt) > 0.5)$

Tweety is a bird. Tweety is a penguin. Therefore, Tweety flies.
$(Pr(Ft \mid Bt \wedge Pt) < 0.5)$

There are formulas φ, ψ, and χ and a stochastic truth table such that

$$Pr(\psi \mid \varphi) > Pr(\psi) \qquad \text{and} \qquad Pr(\psi \mid \varphi \wedge \chi) < Pr(\psi)$$

prob	P	Q	R
0.1	T	T	T
0.25	T	T	F
0.2	T	F	T
0.05	T	F	F
0.05	F	T	T
0.05	F	T	F
0.2	F	F	T
0.1	F	F	F

$$Pr(Q \mid P) = \frac{Pr(Q \wedge P)}{Pr(P)} = \frac{0.35}{0.6} \approx 0.58 > 0.45 = Pr(Q)$$

$$Pr(Q \mid P \wedge R) = \frac{Pr(Q \wedge P \wedge R)}{Pr(P \wedge R)} = \frac{0.1}{0.3} \approx 0.33 < 0.45 = Pr(Q)$$

Summary

- If $\varphi \Rightarrow \psi$ is valid, then $\varphi, \chi \Rightarrow \psi$ is valid.
- If $\varphi \rightarrow \psi$ is a tautology, then $(\varphi \wedge \chi) \rightarrow \psi$ is a tautology.
- There are formulas φ, ψ, and χ, a stochastic truth table and $p \geq 0.5$ such that $Pr(\psi \mid \varphi) > p$ and $Pr(\psi \mid \varphi \wedge \chi) < p$.
- There are formulas φ, ψ, and χ, and a stochastic truth table such that $Pr(\psi \mid \varphi) > Pr(\psi)$ and $Pr(\psi \mid \varphi \wedge \chi) < Pr(\psi)$.