Reasoning for Humans: Clear Thinking in an Uncertain World

PHIL 171

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Kolmogrov Axioms

Given any stochastic truth table, for any formulas X and Y:

- $Pr(X) \geq 0$
- If X is a tautology, then $Pr(X) = 1$
- If X and Y are mutually exclusive, then $Pr(X \vee Y) = Pr(X) + Pr(Y)$

Laws of Probability

In any stochastic truth table, for all X, $Pr(\neg X) = 1 - Pr(X)$

In any stochastic truth table, for all X, if X is a contradiction, then $Pr(X) = 0$

In any stochastic truth table, for all X and Y, if $X \leftrightarrow Y$ is a tautology (i.e., X and Y are tautologically equivalent), then $Pr(X) = Pr(Y)$

Laws of Probability

In any stochastic truth table, for all X and Y,
$Pr(X) = Pr(X \wedge Y) + Pr(X \wedge \neg Y)$

In any stochastic truth table, for all X and Y,
$Pr(X \vee Y) = Pr(X) + Pr(Y) - Pr(X \wedge Y)$

Laws of Total Probability

In any stochastic truth table, for all X and Y,

$$Pr(X) = Pr(Y)Pr(X \mid Y) + Pr(\neg Y)Pr(X \mid \neg Y)$$

Conjunctions and Conditional Probability

In any stochastic truth table, for all X and Y,
$Pr(X \wedge Y) = Pr(X)Pr(Y \mid X)$

In any stochastic truth table, for all X, Y, and Z
$Pr(X \wedge Y \mid Z) = Pr(X \mid Z)Pr(Y \mid X \wedge Z)$

Conditionals in Stochastic Truth Tables

In any stochastic truth table, for all X and Y,
if $X \to Y$ is a tautology, then $Pr(X) \leq Pr(Y)$

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Conditionals in Stochastic Truth Tables

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$$
\begin{aligned}
1 &= Pr(X \rightarrow Y) && \text{(since } X \rightarrow Y \text{ is a tautology)} \\
&= Pr(\neg X \vee Y) && \text{(since } X \rightarrow Y \approx \neg X \vee Y\text{)}
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&= Pr(\neg X) + Pr(Y) - Pr(\neg X \wedge Y) && \text{(probability axiom)}
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Conditionals in Stochastic Truth Tables

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 & = & Pr(\neg X \vee Y) & \text{(since } X \to Y \approx \neg X \vee Y\text{)} \\
 & = & Pr(\neg X) + Pr(Y) - Pr(\neg X \wedge Y) & \text{(probability axiom)} \\
 & = & 1 - Pr(X) + Pr(Y) - Pr(\neg X \wedge Y) & \text{(probability axiom)}
\end{array}
$$

Conditionals in Stochastic Truth Tables

In any stochastic truth table, for all X and Y,
if $X \rightarrow Y$ is a tautology, then $Pr(X) \leq Pr(Y)$

$$\begin{aligned} 1 &= Pr(X \rightarrow Y) && \text{(since } X \rightarrow Y \text{ is a tautology)} \\ &= Pr(\neg X \vee Y) && \text{(since } X \rightarrow Y \approx \neg X \vee Y\text{)} \\ &= Pr(\neg X) + Pr(Y) - Pr(\neg X \wedge Y) && \text{(probability axiom)} \\ &= 1 - Pr(X) + Pr(Y) - Pr(\neg X \wedge Y) && \text{(probability axiom)} \end{aligned}$$

Rearranging the above equation gives:

$$Pr(X) + Pr(\neg X \wedge Y) = Pr(Y).$$

Since $Pr(\neg X \wedge Y) \geq 0$, we have that $Pr(X) \leq Pr(Y)$.

$$\frac{P}{\therefore \quad C}$$

The argument is **deductively valid** when $P \wedge \neg C$ is **impossible** (there is no truth-value function that makes $P \wedge \neg C$ true).

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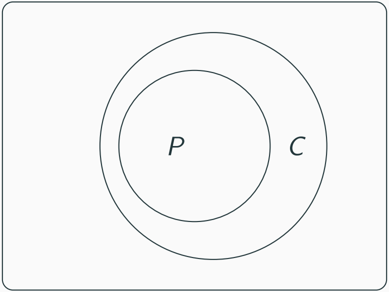

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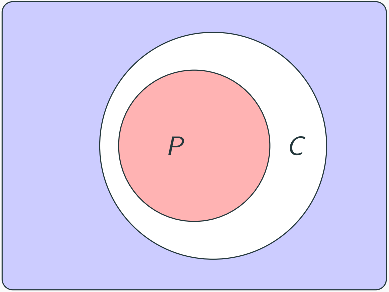


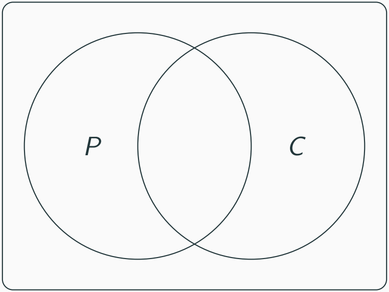
The argument $P \Rightarrow C$ is (deductively) valid when there is no truth value assignment that makes P true and C false

The argument $P \Rightarrow C$ is (deductively) valid when every truth value assignment makes $P \to C$ true

The argument $P \Rightarrow C$ is (deductively) valid when there is no truth value assignment that makes $P \wedge \neg C$ true

$$\frac{P}{\therefore \quad C}$$

The argument is **is inductively strong** if it is **improbable** that C is false while P is true (and it is not deductively valid).



$$\frac{P}{\therefore C}$$

When is an argument inductively strong?

...when it is *improbable* that its conclusion is false while its premises are true.

- when $P \wedge \neg C$ is improbable?
- when $P \rightarrow C$ is probable?

There is a 2000-year-old man in Cleveland.

$\therefore$ There is a 2000-year-old man in Cleveland who has three heads.

This argument is **not** inductively strong, yet:

$P \wedge \neg C$ is improbable because P is improbable.

There is a man in Cleveland who is 1999 years and 11-months-old and in good health.

$\therefore$ No man will live to be 2000 years old.

This argument is **not** inductively strong, yet:

$\neg P \vee C$ is probable because C is probable (so, $P \to C$ is probable).

$$\frac{P}{\therefore C}$$

When is an argument inductively strong?

...when it is *improbable* that its conclusion is false while its premises are true.

- when $P \wedge \neg C$ is improbable? No!
- when $P \rightarrow C$ is probable? No!

The inductive probability of an argument is determined by the evidential relation between its premises and its conclusion, not by the likelihood of the truth of its premises alone or the likelihood of the truth of its conclusion alone.

Every day so far this semester, Ann brought her laptop to class.
Therefore, Ann will bring her laptop to the next class.

Every day so far this semester, Ann brought her laptop to class. Therefore, Ann will bring her laptop to the next class.

Ann said that she saw Bob steal the laptop. Therefore, Bob stole the laptop

Ten eyewitnesses said that they saw Bob steal the laptop. Therefore, Bob stole the laptop.

Conditional Probability

The inductive probability of an argument is the probability that its conclusion is true given that its premises are true.

$$Pr(C \mid P) = \frac{Pr(C \wedge P)}{Pr(P)}$$

There is a 2000-year-old man in Cleveland.

$\therefore$ There is a 2000-year-old man in Cleveland who has three heads.

This argument is **not** inductively strong:

$Pr(C \mid P)$ is low.

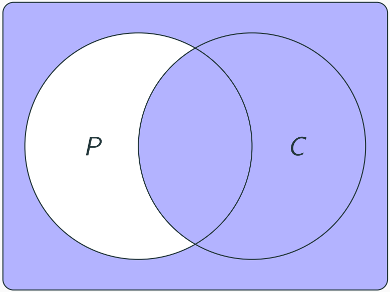
There is a man in Cleveland who is 1999 years and 11-months-old and in good health.

$\therefore$ No man will live to be 2000 years old.

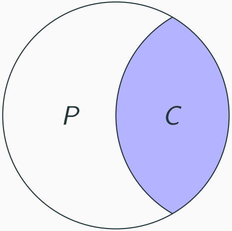
This argument is **not** inductively strong:

$Pr(C \mid P)$ is low.

$$\frac{P}{\therefore\ C}$$



$Pr(P \rightarrow C)$



$Pr(C \mid P)$